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CHANGEPOINTS DETECTION OF PANDEMIC WAVE IN REAL-TIME: APPLICATIONS TO THE TRANSMISSION OF COVID-19

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Abstract. *The COVID-19 pandemic has spread throughout the world. Most countries experienced the pandemic in multiple-waves. The Richards model predicts when a pandemic will peak and end in a particular area. However, this model can only be used in the single-wave case. The research aims to identify a changepoint detection method capable of delineating pandemic wave boundaries, thus enabling the resolution of multiple-wave cases using the Richards model. This article uses two methods to detect changepoints: the Pruned Exact Linear Time (PELT) and the interpolation method. PELT method determines the changepoint based on changes in the statistical properties of the sequence of observations which can be in the form of differences in the mean or variance of each set of observations. In contrast, the linear interpolation method determines the changepoint based on the slope of a data pattern. The two methods complement each other, where the interpolation method is used to determine whether the pandemic is still in a single wave or has multiple-waves, followed by determining wave boundaries using the PELT method. Richards model parameter estimation is carried out after the wave boundaries are obtained, and initial data is taken from the last wave using the PELT method. The prediction results show the peak of the pandemic in a particular region and when it will end, which can be used to inform medium-term strategies for the government to overcome the ongoing pandemic. This information helps prevent a resurgence of infections, which would negatively affect the COVID-19 mortality rate and the area's economic situation.*

Keywords: Changepoint detection method, multiple-wave pandemic, Richards model

1. Introduction

The COVID-19 pandemic has become a global challenge that has seriously affected the lives of people around the world. However, in dealing with a pandemic, it is essential to predict when the pandemic will peak and end in a particular area, especially because many countries face the pandemic in multiple-waves. Although

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many epidemiological models have been developed, most can only predict single-wave pandemics, including the Richards curve model. Therefore, the problem of this research is adapting these models to handle a multiple-wave pandemic.

A multiple-wave pandemic refers to a situation where an infectious disease, such as COVID-19, spreads and causes repeated waves of infection in the population of a region or country. Each wave of infection usually occurs after a decline in new cases, followed by a rebound in the number of new cases over a certain period. In the context of the COVID-19 pandemic, a multiple-wave pandemic can occur due to several factors, including changes in people's behavior, more contagious virus variants, and government policies regarding disease control. Therefore, it is essential to identify and understand these wave patterns to take appropriate action to overcome the pandemic.

COVID-19 is caused by SARS-CoV-2, leading to severe acute respiratory syndrome. COVID-19 has the potential to cause a variety of respiratory issues, ranging from mild symptoms similar to the flu to more severe lung infections such as pneumonia. The initial occurrence of this disease was identified in the Chinese metropolis of Wuhan in late December 2019. The COVID-19 rapidly disseminated among individuals, resulting in infections in numerous countries, including Indonesia, within a few months.

The first case of COVID-19 in Indonesia was detected in Jakarta in early March 2020 until it spread to all parts of Indonesia in mid-2020. Various studies were carried out in order to reduce the rate of spread of COVID-19 such as that of [1] which provides a new approach to estimate the arrival date of an infectious disease based on a small sample that can be applied to many epidemiological situations. The purpose of conducting a COVID-19 study was to monitor the individuals exposed and reduce the overall risk to the community [2]. In South Africa, preventive behavior has evolved. The most frequently utilized preventative strategy is the wearing of masks, which has surpassed handwashing [3]. It is the same with the Netherlands implementing lockdown rules in their country [4]. The government of Indonesia has taken stricter measures, such as physical isolation, home education, and emergency local movement restrictions, to contain the coronavirus outbreak in various regions. This is happening at the same time. Several control measures have been introduced, including strict limits (PKM emergency).

The epidemic model has evolved from the simplest model, namely Susceptible-Infected (SI), Susceptible-Infected-Susceptible (SIS), Susceptible-Infected-Recovered (SIR) [5]. In predicting cases of the COVID-19 pandemic that hit all countries globally, this paper uses the Richards curve model. These two models have been used to describe previous epidemics, including SARS, HIV, and other diseases [6,7,8], and real-time estimates of influenza pandemics [9]. COVID-19 cases are also modeled in different countries, either those that have undergone a single or multiple pandemic wave [10,11]. The obstacle faced by several countries, including Indonesia, is the occurrence of a multi wave pandemic which has caused a surge in positive cases to increase. So that in predicting when the COVID-19 case will end in an area, it is necessary to detect changepoints to determine the starting point for data collection in the last wave. Research has been conducted by [12,13,14] on time series data with multiple changepoints.

This research aims to develop a changepoint detection method that can identify the boundaries of each wave of the COVID-19 pandemic, thereby enabling epidemiological models such as the Richards curve model to predict the development of the pandemic over a more extended period.

This study used two changepoint detection methods: the Pruned Exact Linear Time (PELT) and interpolation for areas experiencing multiple-wave epidemics. We have previously carried out similar research using the binary segmentation method, which was compared with the PELT method [15,16,17,18]. The results obtained by the PELT method are more accurate than the binary segmentation method. We continued this research by adding an interpolation method before applying the PELT method in the multiple-wave case. As an application, data is used from several provinces in Indonesia which are experiencing a multiple-wave epidemic. The best method is then used to determine initial data that will be used to predict when the COVID-19 pandemic will end in several provinces in Indonesia using the Richards curve model.

Most Indonesian provinces were afflicted with the COVID-19 pandemic until the start of 2022. The application of the Richards curve model is limited to forecasting a singular wave of a pandemic. A multi wave pandemic requires the use of changepoint detection to identify the boundaries of each wave. The final wave derived from the changepoint detection approach will be utilized to generate forecasts using the Richards curve model.

We use the PELT approach as our initial changepoint detection technique. The PELT method is an estimation technique that identifies the precise point at which the statistical characteristics of a sequence of observations change. To use the PELT approach effectively, it is crucial that the changepoint does not exhibit an outlier and that the magnitude of the data change does not consistently grow since this would yield more precise detection outcomes. The difficulty in conducting multiple changepoint detections is in determining the ideal number and position of changepoints, as the number of potential solutions grows exponentially with the quantity of the data. By contrast, the second approach uses linear interpolation. The interpolation approach solely necessitates the slope of a data pattern, with a fractional amount of this slope being incrementally incorporated for each interpolation point between two neighboring points. Hence, the algorithm requires minimal operations and can interpolate uniform and non-uniform sample data. This is achieved through the utilization of addition operations, with the only requirement being the satisfaction of the coordinates (x_{i+1}, y_{i+1}). Identifying interpolation points gets increasingly challenging when dealing with noisy data points. Consequently, filters are employed to eliminate noise and identify those locations effectively.

The fundamental difference between the two methods we use is that the PELT method determines the changepoint based on changes in the statistical properties of the sequence of observations which can be in the form of differences in the mean or variance of each set of observations. In contrast, the linear interpolation method determines the changepoint based on the slope of a data pattern. The PELT method and linear interpolation were chosen because both have advantages. PELT can efficiently detect statistical changes in time series data, while linear interpolation is suitable for dealing with non-uniform data. The leading indicators to compare these two methods are the accuracy in detecting changepoints and data processing speed.

The results of this research are expected to provide a better understanding of the dynamics of the COVID-19 pandemic and provide an effective tool for decision-makers to plan appropriate mitigation measures. It is hoped that this will reduce the negative impact of the pandemic on society and the economy. This research also contributes to using changepoint detection methods in the Richards curve model.

2. Matherials and Methods

2.1. *Data*

This research uses COVID-19 data in Indonesia, sourced from https://covid19.go.id/peta-sebaran. This website presents data on confirmed cases of COVID-19 every day from all provinces in Indonesia. The data used starts with detecting the first case of COVID-19 in each province on the island of Java until July 31, 2021. The reason for choosing provinces on the island of Java is because they have experienced multiple-waves of the pandemic. The provinces include West Java, DKI Jakarta, Central Java, East Java, Banten, and DI Yogyakarta.

2.2. *Methods*

Changepoint analysis is the discovery of places within a data collection where the statistical features change. Suppose that we have an ordered series of data, $y_{1:n} = (y_1, y_2, \cdots, y_n)$. Our model will include a number of changepoints, m, as well as their locations, $\tau_{1:m} = (\tau_1, \tau_2, \cdots, \tau_m)$. An often employed method to detect numerous changepoints is to minimize:

$$\sum_{i=1}^{m+1} [\mathcal{C}(y_{(\tau_{i-1}+1):\tau_i})] + \beta f(m), \tag{2.1}$$

where $\mathcal{C}$ is a cost function for a segment, while $\beta f(m)$ is a penalty against overfitting. Both of these functions are used in this context. In the literature on changepoints, a cost function that is frequently utilized is known as twice the negative log-likelihood [19]. The PELT method algorithm is also used by [20]. Steps for the PELT method:

(1) Initialization: Start with the observed time series data and determine the starting point.
(2) Segmentation: Dividing data into segments that may contain points of change.
(3) Statistical Calculations: Calculates relevant statistics for each segment, such as deviation or variation.
(4) Pruning: Uses a pruning algorithm to remove segments unlikely to contain changepoints based on specific criteria, such as low deviation.
(5) Identify change points: Determine change points by comparing statistics in each remaining segment.

The second method used is the interpolation method. To recognize turning locations, we first establish the slopes, S_i, between neighboring interpolation points.

These slopes are then utilized to assess whether or not there is a turning point. We use two criteria in this case. The first criteria determines if the slopes change sign in such a way that a local/absolute minimum or maxima occurs. However, this requirement alone is insufficient to catch all turning points, thus there may be curvatures on times when neighboring slopes do not change sign, such as during S_3. As a result, a second condition is necessary to ensure that these turning locations are properly recognized even when the slopes do not change sign.

As with any other approach, detecting interpolation points becomes more difficult under noisy settings. As a result, filters were used to crudely eliminate noise artifacts and detect these spots, as previously described [21]. Once these sites are identified, turning point criteria concentrate on decreasing random mistakes caused by the interpolation process. The method uses linear interpolation to boost computing efficiency when turning point identification is unavailable.

$$\text{Condition } 1 \rightarrow S_{i-1} > 0 \text{ and } S_i < 0,\; S_{i-1} < 0 \text{ and } S_i > 0, \tag{2.2}$$

$$\text{Condition } 2 \rightarrow \frac{3}{4}|S_{i-1}| > |S_i|,\; \frac{3}{4}|S_i| > |S_{i-1}|. \tag{2.3}$$

Steps for interpolation method:

(1) Data preparation: Prepare time series data to be interpolated.
(2) Mathematical modeling: Choose an appropriate mathematical model to describe the data pattern, such as a linear or polynomial model.
(3) Coefficient calculation: Calculate the coefficients of the mathematical model used for interpolation.
(4) Interpolation: Using a mathematical model and previously calculated coefficients to estimate values between available data points.

2.3. *Prediction Models*

The Richards model is usually solved numerically because proving the analytical solution of the Richards model involves several complex mathematical steps. [22] states that the Richards model is a more comprehensive and detailed variant of the logistic growth model. Within the framework of the logistic growth model, the differential equation is

$$D'(t) = rD(t)\left[1 - \frac{D(t)}{N}\right]. \tag{2.4}$$

The analytical solution of the equation (2.4) is the logistic growth model equation, namely:

$$D(t) = \frac{N}{1 + be^{-rt}}, \tag{2.5}$$

where b is a constant value that depends on the initial conditions, the equation (2.5) is obtained through the following steps.

$$\frac{dD(t)}{dt} = \frac{rD(t)(N - D(t))}{N},$$

$$\frac{N}{D(t)(N - D(t))}dD(t) = rdt.$$

Both sections are integrated:

$$\int \frac{N}{D(t)(N-D(t))} dD(t) = \int r dt, \tag{2.6}$$

by using partial fraction decomposition, it is obtained then the equation (2.6) becomes:

$$\frac{N}{D(t)(N-D(t))} = \frac{1}{D(t)} + \frac{1}{N-D(t)},$$

then the equation (2.6) becomes:

$$\begin{aligned}
\int \left(\frac{1}{D(t)} + \frac{1}{N-D(t)} \right) dD(t) &= \int r dt, \\
\ln |D(t)| - \ln |N-D(t)| &= rt + B, \\
\ln \left| \frac{D(t)}{N-D(t)} \right| &= rt + B, \\
e^{\ln \left| \frac{D(t)}{N-D(t)} \right|} &= e^{rt+B}, \\
\frac{D(t)}{N-D(t)} &= e^{rt} e^{B}.
\end{aligned}$$

It is defined that $e^B = B$, which is a constant, then we obtain:

$$\frac{D(t)}{N-D(t)} = Be^{rt}. \tag{2.7}$$

The equation (2.7) can be changed to:

$$\begin{aligned}
D(t) &= Be^{rt}(N - D(t)), \\
D(t) &= NBe^{rt} - D(t)Be^{rt}, \\
D(t) + D(t)Be^{rt} &= NBe^{rt}, \\
D(t)\left(1 + Be^{rt}\right) &= NBe^{rt}, \\
D(t) &= \frac{NBe^{rt}}{1 + Be^{rt}}. \qquad (2.8)
\end{aligned}$$

If $t = 0$ in the equation (2.7), then we get:

$$\begin{aligned}
\frac{D(0)}{N-D(0)} &= Be^{r(0)}, \\
B &= \frac{D(0)}{N-D(0)}. \qquad (2.9)
\end{aligned}$$

Substitute the equation (2.9) into the equation (2.8), thus obtaining:

$$\begin{aligned}
D(t) &= \frac{N\left(\dfrac{D(0)}{N-D(0)}\right)e^{rt}}{1+\left(\dfrac{D(0)}{N-D(0)}\right)e^{rt}}, \\
&= \frac{ND(0)e^{rt}}{(N-D(0))+D(0)e^{rt}}, \\
&= \frac{Ne^{rt}}{\left(\dfrac{N-D(0)}{D(0)}\right)+e^{rt}}, \\
&= \frac{Ne^{rt}}{b+e^{rt}}, \\
D(t) &= \frac{N}{1+be^{-rt}},
\end{aligned}$$

with basic knowledge of limits, it can be seen that any constant b is $\lim_{t\to\infty} D(t) = N$, as long as N and b are positive. In other words, as long as the total number of epidemic cases and the constant are positive, whatever the population size is at $t = 0$, the population will reach its maximum number as time passes. Thus, the total number of epidemic cases N can be considered the equilibrium value for the logistic model.

This is an addition to the logistic growth model called the Richards model. It adds a new parameter called a to measure how far the curve is from being symmetrical [23]. The differential equation governing the Richards model is:

$$D'(t) = rD(t)\left[1-\left(\frac{D(t)}{N}\right)^{a}\right]. \tag{2.10}$$

The Richards model has an analytical solution, which may be found in [8].

$$D(t) = \frac{N}{[1+e^{-r(t-t_m)}]^{1/a}}, \qquad t_m = t_i + \frac{\ln a}{r}, \tag{2.11}$$

where N represents the total number of cases in the epidemic, r represents the rate at which the infected population is growing, a represents the exponent that measures the deviation from the typical logistic curve, t represents time, and t_i represents the inflection point.

The final step was to test the validity of the estimation results using Mean Absolute Percentage Error (MAPE). MAPE calculates the average percentage difference between observed values and predicted values. This is very useful for assessing the accuracy of predictions relative to the magnitude of actual values. The MAPE value can be calculated using equation (2.12).

$$MAPE = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{y_i-\hat{y}_i}{y_i}\right| \times 100\%. \tag{2.12}$$

3. Results

The initial step that needs to be taken is to determine the number of confirmed cases of COVID-19 in each province selected as a sample. The aim of doing this is to identify the number of pandemic waves that occurred in each province. Figure 1 presents a data plot for each province based on time. The blue curve represents the actual data, the red curve represents the interpolation method applied to the data, and the black line represents the changepoint boundary created based on the PELT method. In addition, the initial data used to make predictions regarding the final wave can be observed in Figure 1.

Table 1. Changepoint results from the PELT method.

Province	Final wave limit	Date
West Java	463	June 15, 2021
DKI Jakarta	472	June 16, 2021
Central Java	457	June 12, 2021
East Java	471	June 30, 2021
Banten	490	July 11, 2021
DI Yogyakarta	476	July 16, 2021

Figure 1 shows that of the six provinces sampled; only Central Java has not shown a decrease in cases in the second wave. From the Figure 1, it is found that the interpolation method is only effective for determining the number of pandemic waves that occur in each province. On the other hand, the PELT approach indicates the number of pandemic waves that have occurred and establishes an initial threshold for data gathering, which may be utilized to forecast the timing of the pandemic's peak and end.

Table 2. Results of Richards model parameter estimates.

Province	N	r	a	t_m
West Java	403,900	0.0632	0.2449	9.58
DKI Jakarta	395,200	0.1077	0.5657	17.85
Central Java	283,500	0.0512	0.3488	19.14
East Java	179,200	0.1043	0.1916	4.41
Banten	60,670	0.1881	0.2126	0.20
DI Yogyakarta	48,500	0.1529	0.3196	2.59

As depicted in Figure 2, most provinces experienced the highest pandemic point in July 2021. The non-linear approach was used to estimate parameters for the six areas modeled using the Richards curve, as shown in Table 2. As seen in Figure 1, the initial assumptions were obtained from the data on COVID-19 instances reported in the provinces. The initial assumption value is determined using different data for each province, ranging from the initial limit specified in Table 1 until July 31, 2021. We utilized the non-linear least squares method to estimate the parameters using

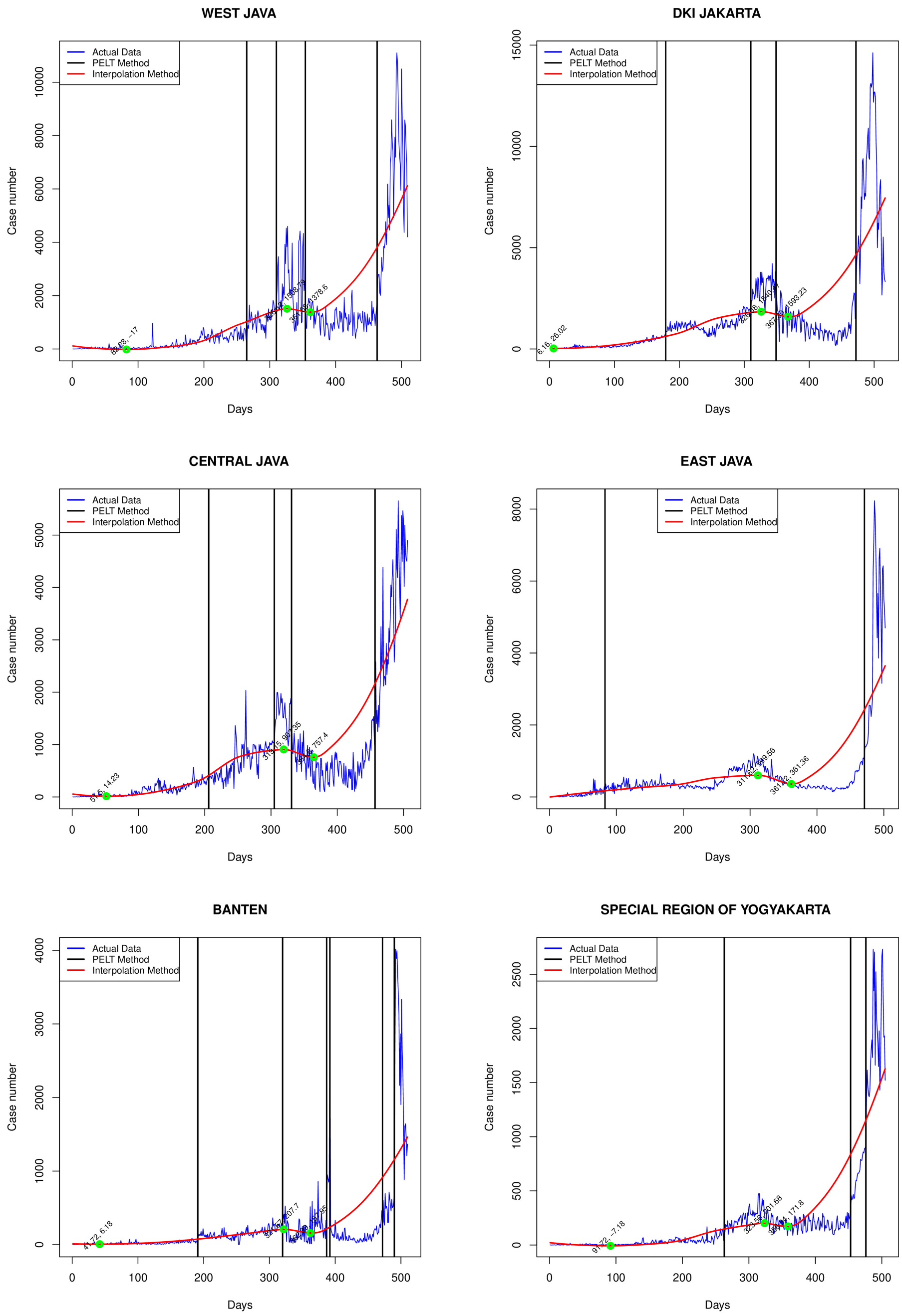

Figure. 1. Comparison of changepoints with the PELT method and interpolation method.

the initial parameters. Once the parameters of each model are obtained, forecasts are generated for each province. The forecast outcomes utilizing the Richards curve are displayed in Figure 2.

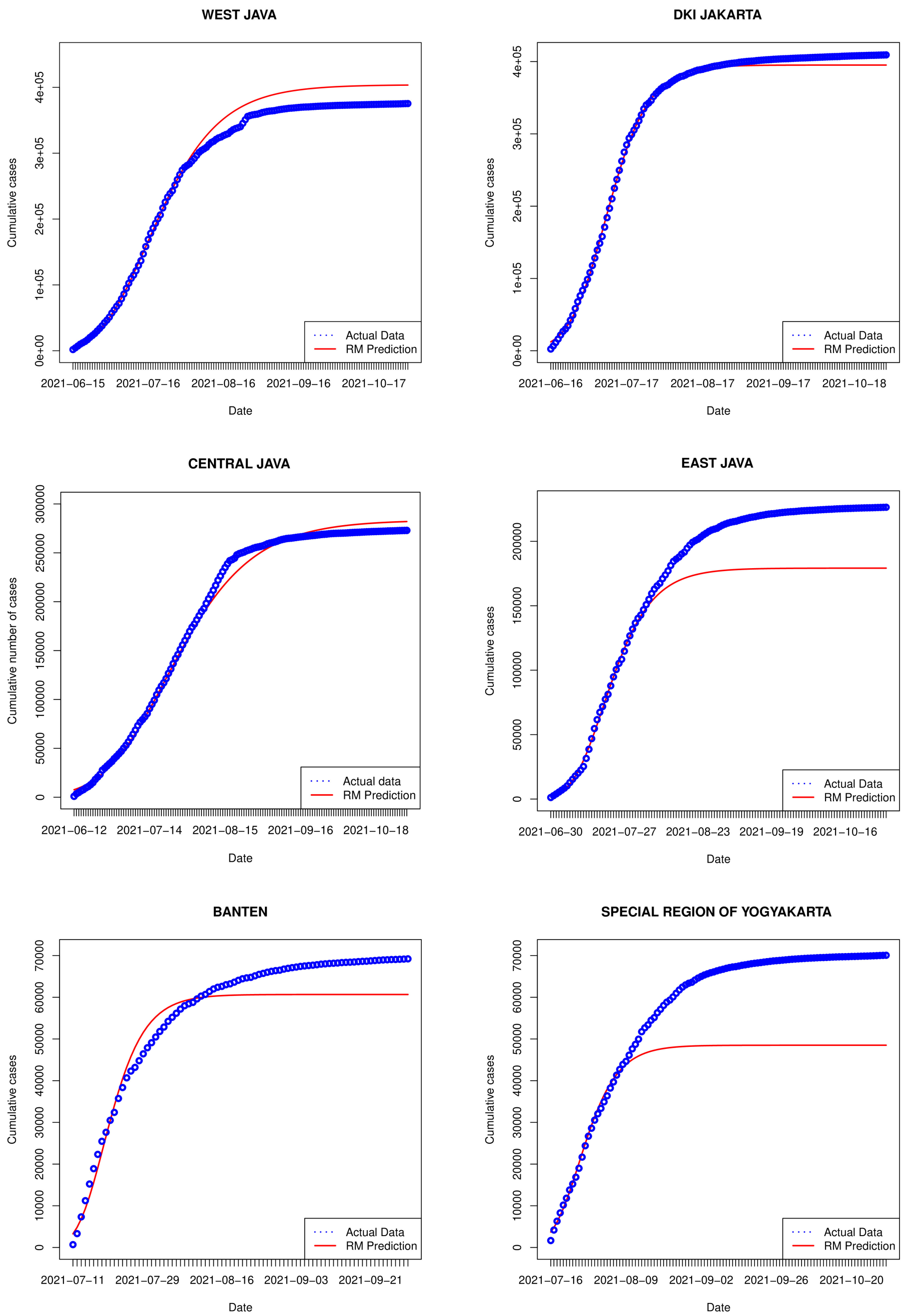

Figure. 2. Prediction results with Richard's curve compared to data up to October 31, 2021.

The Richards curve parameters were estimated in Table 2 to determine the end of the pandemic in a country and to estimate the maximum number of cases in each province. These values were then substituted into equation (2.11). The K value in

Table 2 represents the highest COVID-19 instances recorded in each province. The prediction findings, displayed in Table 3, provide information on the projected end of the pandemic and the highest number of cases that will occur in the province during the outbreak's peak.

Table 3. Prediction results.

Province	End of pandemic	Max. number of cases	MAPE
West Java	December 2021	730,000-740,000	8.07 %
DKI Jakarta	November 2021	840,000-850,000	6.33 %
Central Java	January 2022	490,000-500,000	11.25 %
East Java	October 2021	350,000-360,000	13.84 %
Banten	September 2021	120,000-130,000	13.44 %
DI Yogyakarta	October 2021	130,000-140,000	21.85 %

In this work, we take the province of Java as a sample of the Indonesian province. In states with multi waves, the PELT approach is a helpful tool for determining changepoints and providing information about relevant data to predict the final wave of the pandemic. Initial details of the last wave for each multi wave state based on changepoints in Figure 1 can be found in Table 1.

The information for creating Figure 2 continues until October 31, 2021, following the completion of the PELT method as described in Table 1. This means the consistency of his prediction results for the next 11 days after the prediction was made. Figure 2 The result is close to the Richards curve pattern. After estimating the parameters using nonlinear least squares, the actual data is input into each model equation, and the results can be seen in Figure 2.

Each province after the epidemic has ended is displayed in Table 3 respectively. If the curve does not slope, the rate at which the COVID-19 sickness is spreading will continue to intensify. It is clear from the six data plots that the province of Central Java is the only one that continues to exhibit considerable growth in the number of positive cases. In provinces with many waves, the actual data from the most recent wave is utilized. This changepoint is indicated by the PELT method, as seen in Figure 1 below. As seen in Figure 2, the parameters' estimation findings that were performed using the Richards curve approach are utilized to create predictions.

Approximately in July 2021, the apex of the pandemic will occur in every province, according to reference Figure 2. Four hundred thousand people live in the DKI Jakarta region, making it the region with the most significant number of confirmed positive cases of COVID-19. On the other hand, the province of DI Yogyakarta has the fewest cases of the disease. In addition, the forecasts regarding the termination of the COVID-19 pandemic are shown in Table 3. Central Java is the province experiencing the longest epidemic duration, expected to conclude in January 2021. Additionally, the pandemic in Banten province is also nearing its end. The generated prediction results will be contingent upon the data utilized [24]. Furthermore, government measures are crucial in determining the conclusion of the COVID-19 pandemic. The lower the MAPE value, the better the model prediction quality. In this context, predictions for DKI Jakarta have the lowest MAPE (6.33 %), indicating a higher accuracy level than other provinces. Meanwhile, DI Yogyakarta has the highest MAPE (21.85 %), indicating a lower level of accuracy.

However, in COVID-19, the prediction outcome may diverge from the expected result due to significant uncertainty [25]. The primary sources of uncertainty are the unpredictable social factors and natural disasters that lead to large gatherings of people, resulting in more infected individuals. Additionally, uncertainties arise from a need for more understanding regarding specific events such as hospital capacity, daily testing rates, holidays, and other social factors. Our observation revealed a positive correlation between population density and dispersion width. Consequently, every prediction fails to match its value or yield consistent outcomes.

4. Conclusions

The Richards curve model can be predicted even in an area experiencing multiple-wave pandemics. It is critical to ascertain the changepoint of each pandemic wave in the event of a multi-wave outbreak. The two methods for determining changepoints in this paper complement each other, where the interpolation method is used to determine the presence of new waves that will be predicted. The PELT method is used to determine each wave's boundaries so that each wave's boundaries are used as the basis for collecting initial data to make predictions for the last wave.

The predicted provinces with the most confirmed positive cases of COVID-19 are DKI Jakarta, with 400,000, and Banten province, with the least. Furthermore, predictions are being made for the end of the coronavirus pandemic. The area with the most extended pandemic period is Central Java, which is expected to end in January 2022, and Banten province is also expected to complete its pandemic soon. The predicted results depend on the data used to determine the original assumptions. Furthermore, government intervention in each region will affect the increase or decrease in the number of confirmed positive cases of COVID-19, which will affect the determination of the end of the epidemic in the region.

The implication of this research is the development of a changepoint detection method that can be used to predict the development of the COVID-19 pandemic in the future. This research also provides a better understanding of the factors that influence the spread of the disease and helps in planning effective mitigation measures.

This research's limitations include the data used, especially regarding the availability of complete and accurate data. Additionally, predictions produced by models can be influenced by factors that are difficult to predict, such as changes in government policy or societal behavior. More in-depth research can be carried out on the factors that influence the spread of COVID-19, including further analysis of the effectiveness of different mitigation measures. This research contributes to providing an effective tool for policymakers to plan an appropriate response to the COVID-19 pandemic and reduce its negative impact on society and the economy. Thus, this research can provide valuable insights for policymakers and researchers to control the spread of COVID-19 and protect public health.

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Bibliography

[1] D. L. Roberts, J. S. Rossman, and I. Jari, 2021, Dating first cases of COVID-19, *PLoS Pathogens* Vol. **17**(6): p. e1009620, doi: 10.1371/journal.ppat.1009620.

[2] R. Pung et al., 2020, Investigation of three clusters of COVID-19 in Singapore: implications for surveillance and response measures, *The Lancet* Vol. **395**(10229): 1039 – 1046, doi: 10.1016/S0140-6736(20)30528-6.

[3] U. Kollamparambil and A. Oyenubi, 2021, Behavioural response to the Covid-19 pandemic in South Africa, *PLoS One* Vol. **16**(4): p. e0250269, doi: 10.1371/journal.pone.0250269.

[4] K. Meier et al., Public perspectives on protective measures during the COVID-19 pandemic in the Netherlands, Germany and Italy: A survey study, *PLoS One* Vol. **15**(8): p. e0236917, 2020, doi: 10.1371/journal.pone.0236917.

[5] Z. Y. He, A. Abbes, H. Jahanshahi, N. D. Alotaibi, and Y. Wang, 2022, Fractional-Order Discrete-Time SIR Epidemic Model with Vaccination: Chaos and Complexity, *Mathematics* Vol. **10**(2), doi: 10.3390/math10020165.

[6] X. Luo, H. Duan, and K. Xu, 2021, A novel grey model based on traditional Richards model and its application in COVID-19, *Chaos, Solitons, and Fractals* Vol. **142**, doi: 10.1016/j.chaos.2020.110480.

[7] A. E. Laher et al., 2022, Development and internal validation of the HIV In-hospital Mortality Prediction (HIV-IMP) risk score, *HIV Medicine* Vol. **23**(1), doi: 10.1111/hiv.13159.

[8] Y. H. Hsieh, 2018, Dengue outbreaks in Taiwan, 1998-2017: Importation, serotype and temporal pattern, *Asian Pacific Journal of Tropical Medicine* Vol. **11**(8), doi: 10.4103/1995-7645.240081.

[9] E. Shim, A. Tariq, W. Choi, Y. Lee, and G. Chowell, 2020, Transmission potential and severity of COVID-19 in South Korea, *International Journal of Infectious Diseases* Vol. **93**, doi: 10.1016/j.ijid.2020.03.031.

[10] M. Aghaali, G. Kolifarhood, R. Nikbakht, H. M. Saadati, and S. S. Hashemi Nazari, 2020, Estimation of the serial interval and basic reproduction number of COVID-19 in Qom, Iran, and three other countries: A data-driven analysis in the early phase of the outbreak, *Transboundary and Emerging Diseases* Vol. **67**(6), doi: 10.1111/tbed.13656.

[11] R. Nikbakht, M. R. Baneshi, and A. Bahrampour, 2018, Estimation of the basic reproduction number and vaccination coverage of influenza in the United States (2017-18), *Journal of Research in Health Sciences* Vol. **18**(4): 1 – 6, doi: 10.4103/jrms.JRMS_888_18.

[12] X. Shi, C. Gallagher, R. Lund, and R. Killick, 2022, A comparison of single and multiple changepoint techniques for time series data, *Computational Statistics and Data Analysis* Vol. **170**, doi: 10.1016/j.csda.2022.107433.

[13] R. Baranowski, Y. Chen, and P. Fryzlewicz, 2019, Narrowest over threshold

detection of multiple changepoints and changepoint like features, *Journal of the Royal Statistical Society. Series B: Statistical Methodology* Vol. **81**(3), doi: 10.1111/rssb.12322.

[14] L. Ma, A. J. Grant, and G. Sofronov, 2020, Multiple changepoint detection and validation in autoregressive time series data, *Statistical Papers* Vol. **61**(4): 1507 - 1528, doi: 10.1007/s00362-020-01198-w.

[15] F. Zuhairoh and D. Rosadi, 2020, "Real-time forecasting of the COVID-19 epidemic using the richards model in South Sulawesi, Indonesia," *Indonesian Journal of Science and Technology* Vol. **5**(3): 456 - 462, doi: 10.17509/ijost.v5i3.26139.

[16] F. Zuhairoh and D. Rosadi, 2022, "Data-driven Analysis and Prediction of COVID-19 Infection in Southeast Asia using A Phenomenological Model," *Pakistan Journal of Statistics and Operation Research* Vol. **18**(1): 59 - 69, doi: 10.18187/pjsor.v18i1.3714.

[17] F. Zuhairoh and D. Rosadi, 2022, "Real-time prediction for multi-wave COVID-19 outbreaks," *Communications for Statistical Applications and Methods* Vol. **29**(5): 499 - 512, doi: 10.29220/CSAM.2022.29.5.499.

[18] F. Zuhairoh, D. Rosadi, and A. R. Effendie, 2023, "Prediction of Disease Outbreaks with the SIR Model and Richards Model in Multi-wave Cases," in *AIP Conference Proceedings* Vol. **2975**: 1 - 8, doi: 10.1063/5.0181069.

[19] P. Fearnhead and G. Rigaill, 2019, "Changepoint Detection in the Presence of Outliers," *Journal of the American Statistical Association* Vol. **114**(525), doi: 10.1080/01621459.2017.1385466.

[20] S. Wang and F. Fan, 2021, "Analysis of the response of long-term vegetation dynamics to climate variability using the pruned exact linear time (Pelt) method and disturbance lag model (dlm) based on remote sensing data: A case study in guangdong province (china)," *Remote Sensing* Vol. **13**(10), doi: 10.3390/rs13101873.

[21] M. Aqil, A. Jbari, and A. Bourouhou, 2021, "Comparison of ECG Baseline Wander Removal Techniques and Improvement Based on Moving Average of Wavelet Approximation Coefficients," *International Journal Bioautomation* Vol. **25**(2), doi: 10.7546/ijba.2021.25.2.000770.

[22] M. Maleki, M. R. Mahmoudi, D. Wraith, and K. H. Pho, 2020, Time series modelling to forecast the confirmed and recovered cases of COVID-19, *Travel Medicine and Infectious Disease* Vol. **37**(101742), doi: 10.1016/j.tmaid.2020.101742

[23] K. Roosa et al., 2020, Short-term Forecasts of the COVID-19 Epidemic in Guangdong and Zhejiang, China: February 1323, 2020, *Journal of Clinical Medicine* Vol. **9**(2): 596 - 604, doi: 10.3390/jcm9020596

[24] U. Avila-Ponce de León, Á. G. C. Prez, and E. Avila-Vales, 2020, An SEIARD epidemic model for COVID-19 in Mexico: Mathematical analysis and state-level forecast, *Chaos, Solitons, and Fractals* Vol. **140**, p. 110165, doi: 10.1016/j.chaos.2020.110165

[25] K. C. Santosh, 2020, COVID-19 Prediction Models and Unexploited Data, *Journal of Medical Systems*, Vol. **44** (170): 1 – 4, doi: 10.1007/s10916-020-01645-z