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SIMPLEX TRANSPORTATION METHOD FOR DETERMINING TRANSSHIPMENT OF CLOTHING RAW MATERIALS

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Abstract. *The transshipment problem is a specific issue within transportation. This article discusses the transshipment problem with fuzzy costs. In shipping, unexpected events often occur due to natural conditions, making predetermined costs and times unclear. Therefore, a transshipment model with fuzzy costs is necessary. To find the optimal solution, costs in fuzzy number form are converted into crisp values using robust ranking, then an initial basic solution is found using Vogel approximation method, followed by an optimality test on the initial solution using the simplex transportation method. The optimal solution obtained is then converted back into fuzzy numbers, resulting in costs expressed in fuzzy form. This research yields results with minimum costs, concluding that robust ranking, Vogel approximation method, and the simplex transportation method are effective in seeking optimal solutions in fuzzy cost transshipment models.*

Keywords: Fuzzy Costs, Optimal Solution, Robust Ranking

1. Introduction

Operations research solves problems by producing the optimum value of a decision problem in conditions of limited resources. Transportation is a process of man or goods moving from one place to another using a tool help, land vehicle, oceanic vehicle, and air vehicle, well familiar and also person by use of machine or not utilize machine [1]. Hasbiyati et al. have previously researched transportation problems in various problems ([2], [3], [4]). Part of the transportation problem, namely the route and scheduling problem, can be seen in the research conducted by Setiowati et al. [5]. The transportation problem refers to a unique form of linear programming dealing with shipping from various sources to various destinations. The transportation problem is a classic problem in operations research that involves finding the optimal way

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to move goods from one place to another [6]. Transshipment problem in the standard form is a linear min-cost network flow problem and for such optimization problems [7]. The transportation problem seeks to lower the price of a specific good from several suppliers to a variety of destinations [8]. Transshipment problems can exist in every source and destination when shipping from source to destination makes it possible to transship; that is, goods are sent from several sources to several destinations through other sources and destinations [9]. The transshipment technique is used to find the shortest route from one point in a network to another [10].

Several types of uncertainties are encountered in formulating transshipment problems mathematically due to factors like lack of exact information, unobtainable information, frequent changes in the rate of fuels, traffic jams, or weather conditions [11]. The transportation problem is a linear program that can be solved using the simplex method. Still, due to its unique structure, the simplex transportation method is used, which is computationally much more efficient. This is similar to the discussion, namely, the transshipment problem with the transportation simplex method. The initial basic solution used is the Vogel approximation method, and the optimal solution used the transportation simplex method.

Transshipment is shipping goods involving movement across multiple distribution points before reaching the final destination. In practice, there are challenges caused by cost uncertainty influenced by natural conditions, price fluctuations, and other external factors. This uncertainty makes shipping costs challenging to predict, and traditional transportation models are often less optimal in handling such situations. Therefore, this study applies a transshipment model with a fuzzy approach to address uncertainty in shipping costs.

This study proposes using fuzzy logic to handle uncertain costs in the transshipment model. Using the robust ranking method to convert fuzzy costs into crisp values, the initial solution is determined using Vogel's method, which is subsequently tested for optimality with the simplex transportation method. These steps are expected to produce an optimal transshipment solution even under uncertain cost conditions. The main objective of this study is to develop a transshipment model capable of handling cost uncertainty using a fuzzy approach, aiming to produce a minimum-cost solution. Thus, this research can provide a more reliable solution for shipping scenarios that frequently experience cost fluctuations.

Fuzzy logic has been widely used in decision-making models involving uncertainty, particularly for costs and conditions that are difficult to predict. According to previous research, fuzzy methods can help address cost fluctuations in supply chains. Robust ranking converts fuzzy numbers into crisp values that are easier to analyze. Vogel's method is known to be effective in determining an initial solution to transportation problems, while the simplex transportation method provides optimality assurance for the final solution.

This research is expected to provide transshipment solutions with minimum costs that are more adaptive to cost uncertainty, particularly in the logistics and supply chain industry. The anticipated benefits include increased shipping efficiency and reduced costs in distribution processes that involve high uncertainty.

2. Methods

The costs in fuzzy number form are converted into crisp values using robust ranking. An initial basic solution is found using the Vogel approximation method, followed by an optimality test on the initial solution using the simplex transportation method. The optimal solution obtained is then converted into fuzzy numbers, resulting in costs expressed in fuzzy form.

The following will present some methods of supporting the problem of transshipment of raw materials. The study materials include the Transshipment Problem, Trapezoidal Fuzzy Number, Robust Ranking, Vogel Approximation Method, and Simplex Transportation Method. The following will present some methods of supporting the problem of transshipment of raw materials. The study materials are the Transshipment Problem, Trapezoidal Fuzzy Number, Robust Ranking, Vogel Approximation Method, and Simplex Transportation Method.

2.1. *Transshipment Problem*

The transshipment problem explains that all sources and destinations can give or receive goods from other sources and destinations. In its application, transshipment can occur at the customer's location (other than the origin and destination of the request), at a dedicated transshipment location, or both [12]. Each point can become a transit point for goods or a transshipment point. In the transshipment problem, for example, $i = 1, 2, \cdots, m$, as the source of an item to be transported to destination $j = m+1, m+2, \cdots, m+n$. The sending unit is zero if a sender and receiver have the same point. The coefficient c_{ij} is the cost unit of goods from source i to destination j, and x_{ij} is the amount of goods to be transported from source i to destination j while s_i is the amount supplied at sources i and d_j is a demand at destination j.

In the transshipment problem, each source and destination is seen as a point that can accommodate units of goods distributed for supply or demand. The source and destination require buffer stock (L), which serves to help ensure the availability of goods and anticipate running out of goods to be given to consumers. Each unit of goods moved to each point must be large enough for all transshipment. Units of goods must not exceed the amount produced or received. The total amount of inventory and demand equals L, and L is added at each source and destination point.

$$L = \sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_j = \text{buffer stock}.$$

Furthermore, the mathematical model of the transshipment problem can be reduced to:

$$\min z = \sum_{i=1}^{m+n} \sum_{\substack{j=1 \\ i \neq j}}^{m+n} c_{ij} x_{ij},$$

with constrains:

$$\sum_{j=1}^{m+n} x_{ij} = s_i + L,\ i = 1, 2, \cdots, m,$$

$$\sum_{j=1}^{m+n} x_{ij} = L,\ i = m+1, m+2, \cdots, m+n,$$

$$\sum_{i=1}^{m+n} x_{ij} = L,\ j = 1, 2, \cdots, m,$$

$$\sum_{i=1}^{m+n} x_{ij} = d_j + L,\ j = m+1, m+2, \cdots, m+n,$$

$$x_{ij} \geq 0,\ i, j = 1, 2, \cdots, m+n;\ j = 1, 2, \cdots, m+n;\ c_{ii} = 0.$$

2.2. *Trapezoidal Fuzzy Number*

Fuzzy numbers can be used to solve many problems, such as the transshipment problem, which constrains cost uncertainty. This study uses trapezoidal fuzzy numbers to model uncertainty in costs related to the transshipment problem. Trapezoidal fuzzy numbers are represented by four parameters: the lower limit, the lower peak, the upper peak, and the upper limit. This approach allows for a more flexible handling of uncertainty compared to triangular fuzzy numbers, as it reflects a more realistic range of values for cost and time estimates in uncertain situations.

By using trapezoidal fuzzy numbers, this research aims to achieve a more accurate optimal solution in goods transportation, especially when facing the cost fluctuations that frequently occur in the logistics industry. Using trapezoidal fuzzy numbers, the proposed model is expected to provide more reliable results in decision-making involving variables that cannot be predicted with certainty. The existence of uncertain parameters gives rise to a new problem model called the fuzzy transportation problem [13].

Definition 2.1. *[14] A fuzzy number A is a subset of real line R, with the membership function μA satisfying the following properties:*

(i) $\mu_A(x)$ *is piecewise continuous in its domain.*
(ii) A *is normal, i.e., there is a* $x_0 \in A$ *such that* $\mu_A(x_0) = 1$.
(iii) A *is convex, i.e.,* $\mu_A(\lambda x_1 + (1-\lambda)x_2) \geq \min(\mu_A(x_1), \mu_A(x_2))\, \forall x_1, x_2 \in X$.

Definition 2.2. *[15] The trapezoidal fuzzy number $\tilde{A}$ is a fuzzy number (a, b, c, d) shown in Figure 1, which has the membership function $\mu A(x)$ as following:*

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < a, \\ \dfrac{x-a}{b-a}, & a \leq x \leq b, \\ 1, & b \leq x \leq c, \\ \dfrac{d-x}{d-c}, & c \leq x \leq d, \\ 0, & x \geq d. \end{cases}$$

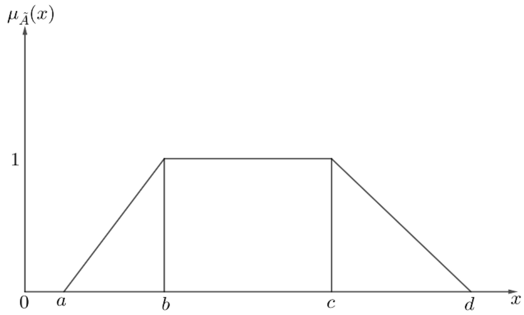

Figure. 1. Trapezoidal curve

2.3. *Robust Ranking*

Assume $\tilde{A} = (a, b, c, d)$ is a trapezoidal fuzzy number. Given the following α-cut: which $\alpha \in [0, 1]$. In the decision making process, the ordering of fuzzy number is very important. Robust ranking method is given to determine the rank of trapezoidal fuzzy number. Robust ranking method is transforming trapezoidal fuzzy number into crisp number. If $\tilde{A} = (a, b, c, d)$ is a trapezoidal fuzzy number, then the robust ranking is as follows [16]:

$$R(A) = \int_0^1 (0.5)(A^l_\alpha, A^u_\alpha)\, d\alpha,$$

where

$$(A^l_\alpha, A^u_\alpha) = ((b - a)\alpha + a, d - (d - c)\alpha)$$

is the α-cut of the trapezoidal fuzzy number. In the case of trapezoidal fuzzy number, the robust ranking becomes as follows [16]:

$$R(\tilde{A}) = \frac{a + b + c + d}{4}.$$

2.4. *Vogel Approximation Method*

The Vogel approximation method deals with finding an optimal solution taking into account the relationships of price indices. This method compares always the two lowest price indices in both a column and a row [17]. Vogel's Approximation Method is known as the best algorithm for generating an efficient initial feasible solution to the transportation problem [18]. To determine the optimal solution of a transportation problem, the first step is to determine the initial basis solution. The initial basis solution can be determine by various methods, one of which is the Vogel approach. The steps of the Vogel approach method, namely:

(i) For each row or column, determine the difference by subtracting the elements' smallest cost in a row or column.

(ii) Then allocate supply with as much demand as possible to the variable with the smallest cost in the selected row and column. Then, readjust supply and demand again and mark the rows or columns that have been fulfilled. If any row and column are filled simultaneously, select one to be marked.

(iii)(a) If a row or column with zero supply or demand is not marked.

(b) If the row or column with supply and demand is not marked, then determine the basis variable in the row or column with the smallest cost.

(c) If all rows and columns that are not marked have supply and demand equal to zero, then determine the basic variable that is zero by choosing the lowest cost and stopping.

2.5. *Simplex Transportation Method*

The transportation simplex method is used because transportation problems are linear programming problems that require many constraints and variables [19]. The computational complexity in the simplex method depends on the number of variables and constraints and is directly proportional to both [20]. The initial basis obtained from solving a problem is not necessarily an optimal solution. The optimal solution is determined by various methods to test and optimize the solution. One way to improve the solution to the transportation problem is to use the simplex transportation method ab, abbreviated as MST.

Let u_i be the dual supply variable and v_j the demand dual variable, with $i = 1, 2, \cdots, m$ and $j = 1, 2, \cdots, n$. The dual from the transportation problem is if

$$\sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_j,$$

then it is said to be balanced. In a balanced transportation problem, the dual form can therefore be written:

$$\min z' = s_1u_1 + s_2u_2 + \cdots + s_mu_m + d_1v_1 + d_2v_2 + \cdots + d_nv_n,$$

with constraints:

$$\begin{gathered}
u_1 + v_1 \leq c_{11}, \\
u_1 + v_2 \leq c_{12}, \\
\vdots \\
u_1 + v_n \leq c_{1n}, \\
u_2 + v_1 \leq c_{21}, \\
u_2 + v_2 \leq c_{22}, \\
\vdots \\
u_2 + v_n \leq c_{2n}, \\
\vdots
\end{gathered}$$

$$u_m + v_1 \leq c_{m1},$$
$$u_m + v_2 \leq c_{m2},$$
$$\vdots$$
$$u_m + v_n \leq c_{mn},$$

where u_i and v_i are independent of signs, for $i = 1, 2, \cdots, m$ and $j = 1, 2, \cdots, n$.

The steps of the simplex transportation method are as follows.

(1) If the question is not balanced, then balance it first.

$$\sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_j.$$

(2) Use one of the methods to find the initial basic feasible solution.

(3) Use the fact that $u_1 = 0$ for all basic variable to find:

$$[u_1, u_2, \cdots, u_m, v_1, v_2, \cdots, v_n],$$

with the equation:

$$u_i + v_j = c_i.$$

(4) Determine the possible costs for each nonbasic variable using the equation:

$$\bar{c}_{ij} = u_i + v_j - c_{ij} \leq 0,$$

with $\bar{c}_{ij}$ is a possible fee.

(5) There are several cases, namely:

(a) If $\bar{c}_{ij}$ is negative, then the solution obtained previously is the optimal solution.

(b) If $\bar{c}_{ij}$ is positive, then the solution obtained is not optimal. Refine the solution by finding variables that will enter the basis (input variables) and variables that will leave the basis (outgoing variables). The input variable is a variable that corresponds to a value of most positive. We construct a closed loop that starts and ends at the input variable to find the input and output variables. The loop consists of successive vertical and horizontal segments in an undefined direction, meaning they can be clockwise or counterclockwise. The exit variable is determined by selecting the smallest base variable in the constructed loop.

Furthermore, the incoming variable is filled in as much as the outgoing variable. For example, suppose the x_{ij} variable is the input variable when x_{ij} is increased by 1 unit to maintain the solution. In that case, the base variable in the loop is adjusted according to the amount of demand and supply by increasing and decreasing the base variable in the loop. The table summarizes this process with (+) and (-) signs in the corresponding cells. The changes obtained will maintain supply constraints and demand constraints to remain fulfilled. Then, return to Step (3).

3. Discussion and Result

A clothing tailor entrepreneur in Rokan Hilir started its business in 1990. Several clothing materials can be selected according to customer requests: Roberto, Maksmara, Songket, and brocade. As time went by, the entrepreneur grew rapidly and opened branches (C), each producing similar goods according to customer requests. If customer demand exceeds the daily capacity, other branches can help produce goods according to the demand that is still lacking. Branch 1 and branch 2 have Production of 30 pcs, and branch 3, branch 4, and branch 5 require 25 pcs, 15 pcs, and 20 additional pcs. The cost of clothing is not yet known for specific reasons, depending on the materials needed. The cost of clothing depends on the materials required by the customer. The materials used are getting more expensive, and vice versa for materials. If the standard is used, then the price is also standard. The transportation cost table with two sources and three destinations can be seen in Table 1. To guarantee each point is capable of according to the total units of goods to be distributed, it needs to be increased even L to the point of supply and demand. Next, determine the supply and demand for buffer stock (L).

$$
\begin{aligned}
L &= \sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_j = \text{Buffer Stock}, \\
&= \sum_{i=1}^{2} s_i = \sum_{j=3}^{5} d_j = 60.
\end{aligned}
$$

Estimation of shipping costs for transshipment problems with cost coefficient trapezoidal fuzzy can be seen in Table 1.

Table 1. Transshipment Problem with Cost Coefficient Fuzzy

	C1	**C2**	**C3**	**C4**	**C5**	**Supply**
C1	$(0;0;0;0)$	$(1;3;3;5)$	$(2;3;5;6)$	$(3;5;6;10)$	$(5;7;8;12)$	$30+L$
C2	$(1;2;2;3)$	$(0;0;0;0)$	$(2;5;8;13)$	$(1;2;6;11)$	$(4;6;10;20)$	$30+L$
C3	$(2;3;6;9)$	$(6;7;9;14)$	$(0;0;0;0)$	$(1;2;4;5)$	$(1;2;5;7)$	L
C4	$(6;10;13;15)$	$(4;5;7;8)$	$(5;8;9;10)$	$(0;0;0;0)$	$(1;1;2;4)$	L
C5	$(2;4;4;6)$	$(1;1;3;3)$	$(2;4;5;9)$	$(9;10;13;16)$	$(0;0;0;0)$	L
Demand	L	L	$25+L$	$15+L$	$20+L$	

Then, substituting $L = 60$ into Table 1, we get the table transportation with trapezoidal fuzzy number cost in Table 2.

The mathematical model of the standard transportation problem can be written

Table 2. Transshipment Problem with Cost Coefficient Fuzzy with $L = 60$

	C1	C2	C3	C4	C5	Supply
C1	$(0;0;0;0)$	$(1;3;3;5)$	$(2;3;5;6)$	$(3;5;6;10)$	$(5;7;8;12)$	90
C2	$(1;2;2;3)$	$(0;0;0;0)$	$(2;5;8;13)$	$(1;2;6;11)$	$(4;6;10;20)$	90
C3	$(2;3;6;9)$	$(6;7;9;14)$	$(0;0;0;0)$	$(1;2;4;5)$	$(1;2;5;7)$	60
C4	$(6;10;13;15)$	$(4;5;7;8)$	$(5;8;9;10)$	$(0;0;0;0)$	$(1;1;2;4)$	60
C5	$(2;4;4;6)$	$(1;1;3;3)$	$(2;4;5;9)$	$(9;10;13;16)$	$(0;0;0;0)$	60
Demand	60	60	85	75	80	

as follows:

$$\begin{aligned}\min z =&(0,0,0,0)x_{11} + (1,3,3,5)x_{12} + (2,3,5,6)x_{13} + (3,5,6,10)x_{14} + (5,7,8,12)x_{15}\\&+ (1,2,2,3)x_{21} + (0,0,0,0)x_{22} + (2,5,8,13)x_{23} + (1,2,6,11)x_{24} + (4,6,10,20)x_{25}\\&+ (2,3,6,9)x_{31} + (6,7,9,14)x_{32} + (0,0,0,0)x_{33} + (1,2,4,5)x_{34} + (1,2,5,7)x_{35}\\&+ (6,10,13,15)x_{41} + (4,5,7,8)x_{42} + (5,8,9,10)x_{43} + (0,0,0,0)x_{44} + (1,1,2,4)x_{45}\\&+ (2,4,4,6)x_{51} + (1,1,3,3)x_{52} + (2,4,5,9)x_{53} + (9,10,13,16)x_{54} + (0,0,0,0)x_{55},\end{aligned}$$

subject to:

$$\begin{aligned}x_{11} + x_{12} + x_{13} + x_{14} + x_{15} &= 90,\\x_{21} + x_{22} + x_{23} + x_{24} + x_{25} &= 90,\\x_{31} + x_{32} + x_{33} + x_{34} + x_{35} &= 60,\\x_{41} + x_{42} + x_{43} + x_{44} + x_{45} &= 60,\\x_{51} + x_{52} + x_{53} + x_{54} + x_{55} &= 60,\\x_{11} + x_{21} + x_{31} + x_{41} + x_{51} &= 60,\\x_{12} + x_{22} + x_{32} + x_{42} + x_{52} &= 60,\\x_{13} + x_{23} + x_{33} + x_{43} + x_{53} &= 85,\\x_{14} + x_{24} + x_{34} + x_{44} + x_{54} &= 75,\\x_{15} + x_{25} + x_{35} + x_{45} + x_{55} &= 80,\end{aligned}$$

where

$$\begin{aligned}&x_{ij} \geq 0, \text{ for } i = 1,2,3,4,5 \text{ and } j = 1,2,3,4,5,\\&\tilde{c}_{ii} = 0, \text{ for } i = 1,2,3,4,5.\end{aligned}$$

The steps to solve the transshipment problem are as follows:

(1) Checking the balance between supply and demand. The amount of inventory can be written as:

$$\sum_{i=1}^{5} s_i = 90 + 90 + 60 + 60 + 60 = 360,$$

and the demand amount:

$$\sum_{j=1}^{5} d_j = 90 + 90 + 60 + 60 + 60 = 360,$$

because supply and demand have the same value, i.e.

$$\sum_{i=1}^{5} s_i = \sum_{j=1}^{5} d_j = 360,$$

then the transshipment problem is balanced.

(2) Changing shipping costs in the form of trapezoidal fuzzy numbers into crisp numbers. Robust ranking is used to change the shipping cost coefficient from fuzzy numbers to crisp numbers of equation (i). The following firm numbers are obtained:

$$R(A) = \frac{a + b + c + d}{4},$$

where

$$\begin{aligned}
\tilde{c}_{11} &= (0, 0, 0, 0), \quad c_{11} = 0,\\
\tilde{c}_{12} &= (1, 3, 3, 5), \quad c_{12} = 3,\\
\tilde{c}_{13} &= (2, 3, 5, 6), \quad c_{13} = 4,\\
\tilde{c}_{14} &= (3, 5, 6, 10), \quad c_{14} = 6,\\
\tilde{c}_{15} &= (5, 7, 8, 12), \quad c_{15} = 8,\\
\tilde{c}_{21} &= (1, 2, 2, 3), \quad c_{21} = 2,\\
\tilde{c}_{22} &= (0, 0, 0, 0), \quad c_{22} = 0,\\
\tilde{c}_{23} &= (2, 5, 8, 13), \quad c_{23} = 7,\\
\tilde{c}_{24} &= (1, 2, 6, 11), \quad c_{24} = 5,\\
\tilde{c}_{25} &= (4, 6, 10, 20), \quad c_{25} = 10,\\
\tilde{c}_{31} &= (2, 3, 6, 9), \quad c_{31} = 5,\\
\tilde{c}_{32} &= (6, 7, 9, 14), \quad c_{32} = 9,\\
\tilde{c}_{33} &= (0, 0, 0, 0), \quad c_{33} = 0,\\
\tilde{c}_{34} &= (1, 2, 4, 5), \quad c_{34} = 3,\\
\tilde{c}_{35} &= (1, 2, 5, 7), \quad c_{35} = 4,\\
\tilde{c}_{41} &= (6, 10, 13, 15), \quad c_{41} = 11,\\
\tilde{c}_{42} &= (4, 5, 7, 8), \quad c_{42} = 6,\\
\tilde{c}_{43} &= (5, 8, 9, 10), \quad c_{43} = 8,\\
\tilde{c}_{44} &= (0, 0, 0, 0), \quad c_{44} = 0,\\
\tilde{c}_{45} &= (1, 1, 2, 4), \quad c_{45} = 2,\\
\tilde{c}_{51} &= (2, 4, 4, 6), \quad c_{51} = 4,
\end{aligned}$$

$$\tilde{c}_{52} = (1, 1, 3, 3), \quad c_{52} = 2,$$
$$\tilde{c}_{53} = (2, 4, 5, 9), \quad c_{53} = 5,$$
$$\tilde{c}_{54} = (9, 10, 13, 16), \quad c_{54} = 12,$$
$$\tilde{c}_{55} = (0, 0, 0, 0), \quad c_{55} = 0.$$

Changes in shipping costs in the from crisp number can be seen in Table 3.

Table 3. Shipping Costs in Crisp Number

	C1	**C2**	**C3**	**C4**	**C5**	**Supply**
C1	0	3	4	6	8	90
C2	2	0	7	5	10	90
C3	5	9	0	3	4	60
C4	11	6	8	0	2	60
C5	4	2	5	12	0	60
Demand	60	60	85	75	80	

(3) Then, using Vogel approach method to find the initial base solution.

(a) Investigate the two smallest values in each row and column. If there are the same values, you can choose one of them. Each row and column is obtained as follows:

$$\text{row } 1 = 3 - 0 = 3,$$
$$\text{row } 2 = 2 - 0 = 2,$$
$$\text{row } 3 = 3 - 0 = 3,$$
$$\text{row } 4 = 2 - 0 = 2,$$
$$\text{row } 5 = 2 - 0 = 2,$$
$$\text{column } 1 = 2 - 0 = 2,$$
$$\text{column } 2 = 2 - 0 = 2,$$
$$\text{column } 3 = 4 - 0 = 4,$$
$$\text{column } 4 = 3 - 0 = 3,$$
$$\text{column } 5 = 2 - 0 = 2.$$

Based on the results of each row and column, select the largest value in column 3, which is 4.

(b) Allocate as much demand and inventory as possible to the variables with the smallest cost in the rows and columns. Then, delete rows and columns that have been exhausted when allocated. The smallest cost element in column 3 is selected, 0 in row 3. Allocated demand and inventory in column 3, row 3, which is 60 pcs. Rows or columns that have been selected are then deleted if they have no inventory or demand. Cost changes can be seen in Table 4.

(c) Re-investigate each row and column. Then allocate the supply and demand until they are all used up (see Table 5).

Table 4. Iteration 1 Vogel Approximation Method

	C1	**C2**	**C3**	**C4**	**C5**	**Supply**
C1	0	3	4	6	8	90
C2	2	0	7	5	10	90
C3	5	9	0 60	3	4	
C4	11	6	8	0	2	60
C5	4	2	5	12	0	60
Demand	60	60	25	75	80	

Table 5. Iteration 2 Vogel Approximation Method

	C1	**C2**	**C3**	**C4**	**C5**	**Supply**
C1	0	3	4	6	8	90
C2	2	0	7	5	10	90
C4	11	6	8	0 60	2	60
C5	4	2	5	12	0	60
Demand	60	60	25	15	80	

In Table 5, each row and column value is obtained, i.e.:

$$\begin{aligned}
\text{row } 1 &= 3 - 0 = 3,\\
\text{row } 2 &= 2 - 0 = 2,\\
\text{row } 4 &= 2 - 0 = 2,\\
\text{row } 5 &= 2 - 0 = 2,\\
\text{column } 1 &= 2 - 0 = 2,\\
\text{column } 2 &= 2 - 0 = 2,\\
\text{column } 3 &= 5 - 4 = 1,\\
\text{column } 4 &= 5 - 0 = 5,\\
\text{column } 5 &= 2 - 0 = 2.
\end{aligned}$$

Based on the results of each row and column, select the largest value in column 4, which is 5. The largest cost element in column 4 is worth 5. In column 4, select the smallest cost, which is 0 in row 4. Then allocate demand and inventory in column 4 and row 4 as much as 60 pcs. Cost changes can be seen in Table 6.

Table 6. Iteration 3 Vogel Approximation Method

	C1	C2	C3	C4	C5	Supply
C1	0	3	4	6	8	90
C2	2	0	7	5	10	90
C5	4	2	5	12	0 60	
Demand	60	60	25	15	20	

In Table 6, each row and column value is obtained, i.e.:

$$\begin{aligned} \text{row } 1 &= 3-0=3, \\ \text{row } 2 &= 2-0=2, \\ \text{row } 5 &= 2-0=2, \\ \text{column } 1 &= 2-0=2, \\ \text{column } 2 &= 2-0=2, \\ \text{column } 3 &= 5-4=1, \\ \text{column } 4 &= 6-5=1, \\ \text{column } 5 &= 8-0=8. \end{aligned}$$

Based on the results of each row and column, select the largest value in column 5, which is worth 8. The cost element in column 5 is selected as the smallest cost, which is worth 0, in row 1. Then allocate demand and inventory in column 5 and row 1 as many as 10 pcs. Cost changes can be seen in Table 7.

Table 7. Iteration 4 Vogel Approximation Method

	C1	C2	C3	C4	C5	Supply
C1	0 60	3	4	6	8	30
C2	2	0	7	5	10	90
Demand		60	25	15	20	

In Table 7, each row and column value is obtained, i.e.:

$$\begin{aligned} \text{row } 1 &= 3-0=3, \\ \text{row } 2 &= 2-0=2, \\ \text{column } 1 &= 2-0=2, \\ \text{column } 2 &= 3-0=3, \\ \text{column } 3 &= 7-4=3, \\ \text{column } 4 &= 6-5=1, \\ \text{column } 5 &= 10-8=2. \end{aligned}$$

Based on the results of each row and column, select the largest value in row 1, column 2, and column 3, because it has 3 equal values, select one of them, namely in row 1, which is worth 3. The cost element in row 1 is selected as the smallest cost, which is worth 0 in column 1. Then allocate demand and inventory in row 1 and column 1 as many as 60 pcs. Cost changes can be seen in Table 8.

Table 8. Iteration 5 Vogel Approximation Method

	C2	**C3**	**C4**	**C5**	**Supply**
C1	3	4	6	8	30
C2	0 60	7	5	10	30
Demand		25	15	20	

In Table 8, each row and column value is obtained, i.e.:

$$\text{row } 1 = 4 - 3 = 1,$$
$$\text{row } 2 = 5 - 0 = 5,$$
$$\text{column } 2 = 3 - 0 = 3,$$
$$\text{column } 3 = 7 - 4 = 3,$$
$$\text{column } 4 = 6 - 5 = 1,$$
$$\text{column } 5 = 10 - 8 = 2.$$

Based on the results of each row and column, select the largest value in row 2, which is worth 5. The cost element in row 2 is selected as the smallest cost, which is worth 0 in column 2. Then allocate demand and inventory in row 2 and column 2 as many as 60 pcs. Cost changes are given in Table 9.

Table 9. Iteration 6 Vogel Approximation Method

	C3	**C4**	**C5**	**Supply**
C1	4 25	6	8	5
C2	7	5	10	30
Demand		15	20	

In Table 9, each row and column value is obtained, i.e.:

$$\text{row } 1 = 6 - 4 = 2,$$
$$\text{row } 4 = 7 - 5 = 2,$$
$$\text{column } 3 = 7 - 4 = 3,$$
$$\text{column } 4 = 6 - 5 = 1,$$
$$\text{column } 5 = 10 - 8 = 2.$$

Based on the results of each row and column, select the largest value in column 3, which is worth 3. The cost element in column 3 is selected as the smallest cost, which is worth 4 in row 1. Then allocate demand and inventory in column 3 and row 1 as many as 25 pcs. Cost changes can be seen in Table 10.

Table 10. Iteration 7 Vogel Approximation Method

	C4	**C5**	**Supply**
C1	6	8	5
C2	5 15	10	15
Demand		20	

In Table 10, each row and column value is obtained, i.e.:

$$\text{row } 1 = 8 - 6 = 2,$$
$$\text{row } 2 = 10 - 5 = 5,$$
$$\text{column } 4 = 6 - 5 = 1,$$
$$\text{column } 5 = 10 - 8 = 2.$$

Based on the results of each row and column, select the largest value in row 2, which is worth 5. The cost element in row 2 is selected as the smallest cost, which is worth 5 in column 4. Then allocate demand and inventory in row 2 and column 4 as many as 15 pcs. Cost changes are given in Table 11.

Table 11. Iteration 8 Vogel Approximation Method

	C5	**Supply**
C1	8	5
C2	10 15	
Demand	5	

In Table 11, each row and column value is obtained, i.e.:

$$\text{row } 1 = 8 = 8,$$
$$\text{row } 2 = 10 = 10,$$
$$\text{column } 5 = 10 - 8 = 2.$$

Based on the results of each row and column, select the largest value in row 2, which is worth 10. The cost element in row 2, which is worth 10, is in column 5. Then allocate demand and inventory in row 2 and column 5 as many as 15 pcs. The cost changes can be seen in Table 12.

Table 12. Iteration 9 Vogel Approximation Method

	C5	Supply
C1	8 5	5
Demand	5	

In Table 12, each row and column value is obtained, i.e.:

$$\text{row } 1 = 8,$$
$$\text{column } 5 = 8.$$

Based on the remaining rows and columns, namely row 1 and column 5, the demand and inventory of 5 units of goods can be directly allocated.

(d) The initial base solution of the transshipment problem is obtained. The initial solution obtained is

$$x_{11} = 60,\ x_{13} = 25,\ x_{15} = 5,$$
$$x_{22} = 60,\ x_{24} = 15,\ x_{25} = 15,$$
$$x_{33} = 60,\ x_{44} = 60,\ \text{and } x_{55} = 60.$$

with total transportation costs are $(150, 230, 405, 640)$. The results of this calculation can be seen in Table 13.

Table 13. Cost Allocation Vogel Approach Method

	C1	C2	C3	C4	C5	Supply
C1	0 60	3	4 25	6	8 5	90
C2	2	0 60	7	5 15	10 15	90
C3	5	9	0 60	3	4	60
C4	11	6	8	0 60	2	60
C5	4	2	5	12	0 60	60
Demand	60	60	85	75	80	

(4) Next, the optimality of the initial base solution is checked using the simplex transport method. The values of u_i and v_j are determined through each base

variable:

$$\begin{aligned} u_i + v_j &= c_{ij}, \\ u_1 &= 0, \quad \text{initial value}, \\ u_1 + v_1 = 0 &\quad \rightarrow \quad v_1 = 0, \\ u_1 + v_3 = 4 &\quad \rightarrow \quad v_3 = 4, \\ u_1 + v_5 = 8 &\quad \rightarrow \quad v_5 = 8, \\ u_2 + v_5 = 10 &\quad \rightarrow \quad u_2 = 2, \\ u_2 + v_4 = 5 &\quad \rightarrow \quad v_4 = 3, \\ u_2 + v_2 = 0 &\quad \rightarrow \quad v_2 = -2, \\ u_3 + v_3 = 0 &\quad \rightarrow \quad u_3 = -4, \\ u_4 + v_4 = 0 &\quad \rightarrow \quad u_4 = -3, \\ u_5 + v_5 = 0 &\quad \rightarrow \quad u_5 = -8. \end{aligned}$$

Furthermore, for each non-basis, $\tilde{c}_{ij}$ is determined, namely:

$$\begin{aligned} &\tilde{c}_{12} = -5, \tilde{c}_{14} = -1, \tilde{c}_{21} = 0, \tilde{c}_{23} = -1, \\ &\tilde{c}_{31} = -9, \tilde{c}_{32} = -15, \tilde{c}_{34} = -2, \tilde{c}_{35} = 0, \\ &\tilde{c}_{41} = -14, \tilde{c}_{42} = -11, \tilde{c}_{43} = -7, \tilde{c}_{45} = 3, \\ &\tilde{c}_{51} = -12, \tilde{c}_{52} = -12, \tilde{c}_{53} = -9, \tilde{c}_{54} = -15. \end{aligned}$$

It can be seen that the non-base value of $\tilde{c}_{45} = 3$ is greater than zero or positive, which means that the initial base solution is not optimal. Furthermore, x_{45} will become the base by forming a loop that starts and ends at x_{45} namely $x_{45}(+) \rightarrow x_{44}(-) \rightarrow x_{24}(+) \rightarrow x_{25}(-) \rightarrow x_{45}(+)$. The exit variable is determined by selecting the smallest cost in the loop which is $\min\{x_{25}, x_{44}\} = \min\{15, 60\} = 15$. Then obtained element changes are $x_{45} = 15$, $x_{44} = 45$, $x_{24} = 30$, and $x_{25} = 0$. Cost reduction can be seen in Table 14.

Table 14. Vogel Approach Method Cost Reduction

	C1	**C2**	**C3**	**C4**	**C5**	**Supply**
C1	0 60	3	4 25	6	8 5	90
C2	2	0 60	7	5 $\rightarrow$ 15	10 $\downarrow$ 15	90
C3	5	9	0 60	3	4	60
C4	11	6	8	0 $\uparrow$ 60	2 $\leftarrow$	60
C5	4	2	5	12	0 60	60
Demand	60	60	85	75	80	

Furthermore, for each non-basis, $\tilde{c}_{ij}$ is determined, namely:

$$\tilde{c}_{12} = -2,\ \tilde{c}_{15} = -13,\ \tilde{c}_{21} = -15,\ \tilde{c}_{24} = -5,\ \tilde{c}_{25} = -13,$$
$$\tilde{c}_{31} = -13,\ \tilde{c}_{32} = -7,\ \tilde{c}_{34} = -13,\ \tilde{c}_{35} = -21,\ \tilde{c}_{41} = -6,$$
$$\tilde{c}_{42} = -7,\ \tilde{c}_{43} = -3,\ \tilde{c}_{51} = -13,\ \tilde{c}_{52} = -11,\ \tilde{c}_{53} = -5,\ \tilde{c}_{54} = -3.$$

It can be seen that all non-bases are smaller than zero or negative. The optimization requirements have been met so that the optimal solution to the transshipment problem is obtained, namely $x_{13} = 25$, $x_{15} = 5$, $x_{24} = 30$, and $x_{45} = 15$. The optimal solution to the transshipment problem can be written in the form of a route as shown in Figure 2.

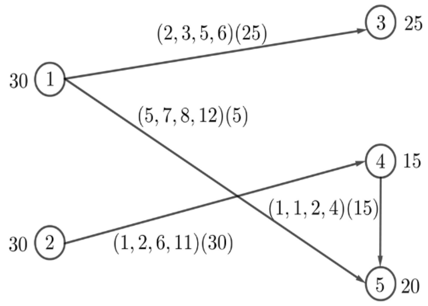

Figure. 2. Transshipment Problem Representation

The total minimum cost of the transshipment problem is:

$$\begin{aligned}\min z &= \tilde{c}_{13}x_{13} + \tilde{c}_{15}x_{15} + \tilde{c}_{24}x_{24} + \tilde{c}_{45}x_{45},\\ &= (2,3,5,6)(25) + (5,7,8,12)(5) + (1,2,6,11)(30) + (1,1,2,4)(15),\\ &= (50,75,125,150) + (25,35,40,60) + (30,60,180,330) + (15,15,30,60),\\ &= (50+25+30+15, 75+35+60+15, 125+40+180+30, 150+60+330+60)\\ &= (120,185,375,600).\end{aligned}$$

Based on the discussion of the fuzzy cost coefficient transshipment problem with the Vogel approach method, the initial base solution is produced. Then to get the optimal solution, the initial base solution optimization test is carried out with the transportation simplex method. The minimum total cost is $(120, 185, 375, 600)$ per pcs.

Transshipment problems can be divided into recurrent problems and non-recurrent problems. For the case of the transshipment problem with the recurrent method using looping, while for the non-recurrent case does not use looping. The example case in this article is the recurrent case, and the looping is shown in Table 14. In the repeated case example, the loop occurs when performing optimality tests on the initial base solution. In the iterative case, the loop occurs when performing optimality tests on the initial base solution. It is the retesting of the base that causes the loop to occur.

The initial base solution obtained through the Vogel method is then tested for optimality using the simplex transportation method. The initial base solution of the transshipment problem is obtained. The initial solution obtained is $x_{11} = 60$, $x_{13} = 25$, $x_{15} = 5$, $x_{22} = 60$, $x_{24} = 15$, $x_{25} = 15$, $x_{33} = 60$, $x_{44} = 60$, and $x_{55} = 60$, with total transportation costs are $(150, 230, 405, 640)$. The optimization requirements have been met so that the optimal solution to the transshipment problem is obtained, namely $x_{13} = 25$, $x_{15} = 5$, $x_{24} = 30$, and $x_{45} = 15$. The minimum cost obtained from the transshipment problem on the specified route is $(120, 185, 375, 600)$/unit, the minimum cost is obtained which is the optimal route determination in the transshipment problem, namely: from the source supplier 1 to distributor 3, then from the source supplier 1 to distributor 5, then the source distributor 2 to distributor 4 to distributor 5, and transshipment is at the distributor 4.

The optimal solution to the transshipment problem can be written in the form of a route and can be seen in Figure 2.

4. Conclusion

This research provides significant insight into the fuzzy transshipment problem. The findings show that the fuzzy transshipment problem can be solved by the simplex method commonly used in transportation problems. First, the fuzzy numbers are converted into firm numbers by the Robut method. The Vogel method finds the initial basis solution, and optimality is tested by the transportation simplex method. The results of this study add valuable understanding to the fuzzy transshipment problem.

The implications of these findings are critical for identifying transportation issues that experience cost and time uncertainties that natural factors may cause. This research suggests the need to anticipate when shipping goods so that no losses occur. This research offers a foundation for improving the understanding of uncertainty problems by changing the anticipation of the uncertainty element.

The scope of this study is determined by certain limitations that affect the interpretation of the study results. In particular, this research is limited by the fuzzy transshipment problem; natural factors cause the fuzzy element. Although these limitations are necessary to manage the research, they may affect the generalizability of the findings.

Future research should build on this by exploring other uncertainty elements using fuzzy numbers and stochastic programming. Stochastic programming topics can deepen understanding and generate new insights. Advanced research like this is significant for the public because it supports and contributes to the public, for example, inter-city transportation problems.

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Bibliography

[1] Hasbiyati I., 2019, Analysis of Multi-Stage Stochastic Optimization Model for Stochastic Transportation Problems, Journal of Transportation Systems Vol. **4**(1): 21 – 25

[2] Hasbiyati I., Putri W., Adnan A., Ahriyati A., Hasriati, A., 2019, Parking Lot Optimization in Parallelogram Using the Concept Area of Rectangular and Right Triangle, *Pure and Applied Mathematics Journal* Vol. **8**(4): 77 – 82, doi: 10.11648/j.pamj.20190804.12

[3] Hasbiyati I., Abdullah M., Salambue R., and Ahriyati A., 2022, Northwest Corner Method for Natural Disaster Notification, *Barekeng J. Ilmu Mat. dan Terap.* Vol. **16**(1): 271 – 280

[4] Putri W., Hasbiyati I., and Gamal, M. D. H., 2019, Optimization of Parking Lot in the Forms of Parallelogram and Right Triangle for Cars and Motorbikes, *Math. Model. Appl.* Vol. **4**(4): 64 – 71

[5] Setiowati I., Hasbiyati I., and Gamal M. D. H., 2017, Scheduling aircraft landing at single runway, Appl. Math. Sci. Vol. **11**(46): 2265 – 2273

[6] Malacky P. and Madlenak R., 2023, Transportation problems and their solutions: literature review, *Transp. Res. Procedia,* Vol. **74**(2022): 323 – 329

[7] Ellaimony E., Ahmed I., Abdelwahed K., Ahmed R., and Khalil M., 2021, Solving the Transshipment Problem with Fuzzy Cost Coefficients, *Int. J. Sci. Adv.*, Vol. **2**(3): 236 – 240

[8] Hemalatha, K., Venkateswarlu, B., 2023, Pythagorean fuzzy transportation problem: New way of ranking for Pythagorean fuzzy sets and mean square approach, *Heliyon* Vol. **9**(10): e20775

[9] Okiemute G. A. and Akpan N. P., 2017, Transshipment Problem and Its Variants: A Review, *Math. Theory Model.* Vol. **7**(2): 19 – 32

[10] Khurana A., 2015, Variants of transshipment problem, *Eur. Transp. Res. Rev.*, Vol. **7**(2): 1 – 19

[11] Kumar A., Chopra R., and Saxena R. R., 2020, An Efficient Algorithm to Solve Transshipment Problem in Uncertain Environment, *Int. J. Fuzzy Syst.* Vol. **22**(8): 2613 – 2624

[12] Wolfinger D. and Gonzalez J. J. S., 2021, The Pickup and Delivery Problem with Split Loads and Transshipment: A Branch-and-Cut Solution Approach, *Eur. J. Oper. Res.* Vol. **289**(2): 470 – 484

[13] Bisht D. C. S. and Srivastava P. K., 2020, Trisectional fuzzy trapezoidal approach to optimize interval data based transportation problem, *J. King Saud Univ.* Vol. **32**, Issue 1: 195 – 199

[14] Panda A. and Pal M., 2015, A study on pentagonal fuzzy number and its corresponding matrices, *Pacific Sci. Rev. B Humanit. Soc. Sci.* Vol. **1**(3): 131 – 139

[15] Savarimuthu J. J. and Abirami P., 2018, Octagonal Fuzzy Number and its Corresponding Matrices, *Int. Journal of Math. and Its Appl.* Vol. **6**(1): 75 – 82

[16] Ahmed N. U. and Khan A. R., 2015, Solution of mixed type transportation

problem: a fuzzy approach, *Buletinul Institutului Politehnic Din Iaşi*, Tomul LXI (LXV), Fasc. **2**: 19 – 32

[17] Hlatka M., Bartuska L., and Lizbet J., 2017, Application of the Vogel Approximation Method to Reduce Transport-logistics Processes, MATEC Web of Conferences, Vol. **134**(4): 00019

[18] Babu A. and Sharif M. A. H., 2020, A heuristic for obtaining better initial feasible solution to the transportation problem, *OPSEARCH* Vol. **57**: 221 – 245

[19] Haryono Y., 2015, Penyelesaian Masalah Model Transportasi Dengan Menggunakan Metode Simpleks, *Transportasi* Vol. **1**(2): 71 – 77

[20] Sarode M. V., 2017, Application of a Dual Simplex method to Transportation Problem to minimize the cost, *Int. J. Innov. Eng. Sci.* Vol. **2**(7): 51 – 55