LEONTIEF MATRIX: BUSINESS MODEL RECOMMENDATION FOR EXPORT COMMODITY OF NORTH SUMATERA

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Abstract. *The open model as the one the application of Leontief model using the explanation of the economy with input-output model. The open model shows the number of productions needed to satisfy an increase in internal and external demand. By using the operation linear algebra operation on ring characteristics, then the production numbers are calculated. This method is applied on the ten product-producing commodities of North Sumatera export demand to find the total production number, while the amount of demand is defined. It shows a solution to the minimization linear program is the solution that will satisfy both internal and external demands of the commodity with minimum inventory level.*

Keywords: Internal and external demand, Linear algebra, Open sector

1. Introduction

After pandemic condition affects the economic growth at some area in Indonesia, specially Sumatera Island and the second most populous island in Indonesia after Java (BPS-Statistics Indonesia) has increasingly movement. This moment as the economic stabilization of the condition of inflicted an economic hardship unprecedented for the modern age. Therefore, the time to economic recovery towards conditions for normalization habitual economic system. As the province with the 4th largest population in Indonesia. Moreover, North Sumatera has succeeded in

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positioning itself as the largest contributor to Sumatera's economic growth with a contribution of 24.35% in the second quarter of 2020, and is the gateway to the Western Region of Indonesia. This connected to North Sumatera having Gross Domestic Product (GDP) on 2021 mentioned Rp 859,87 trillion. The economic structure on the island of Sumatera spatially in 2021 is dominated by several provinces including North Sumatera Province which contributes the largest to the GRDP on the island of Sumatera by 23.37 percent, Riau Province by 22.92 percent; South Sumatera Province at 13.36 percent, and Lampung Province at 10.11 percent (BPS-Statistics Sumatera Utara).

Domestic products present goods and services as a result of economic activities operating in the domestic territory, regardless of whether the factors of production are derived from or owned by the inhabitants of the region, are domestic products of the region, on North Sumatera (BPS-Statistics Sumatera Utara). The real fact shows that part of the factors of production used in production activities in an area come from another region or from abroad, and vice versa the factors of production owned by the inhabitants of that area take part in the production process in other regions or abroad. This related to the export factor of the commodity at North Sumatera flowchart.

Total number of export commodity at North Sumatera shows the decreasing number at 2019 and 2020 at pandemic economic growth with 7678,56 and 8086,22 (Million US$) respectively. This number far from the data export at 2021 have 11.874,57 (Million US$) total export of North Sumatera (BPS-Statistics Sumatera Utara). This data represents after pandemic situation, the economic growth indicates the stabilization number production of the commodity for exporting fulfilment.

Another focus economic growth also concerning on export number of some commodity to produce the goods and services on North Sumatera. The economy of North Sumatera in the fourth quarter of 2021 against the fourth quarter of 2020 experienced a growth of 3.81 percent (y-on-y). In terms of expenditure, the highest growth in the Export component of Goods and Services was 13.71 percent. Then for the economy of North Sumatera until the fourth quarter of 2021 experienced a growth of 2.61 percent (c-to-c). In terms of expenditure, the highest growth occurred in the Export Component of Goods and Services by 15.09 percent. Therefore, the factor of export commodity takes the main rules for economic growth at North Sumatera (BPS-Statistics Sumatera Utara).

Given this research, we would calculate the mathematical approach using Leontief Economic Model for predicting total number each commodity in North Sumatera. This method is chosen on this case based on the Leontief Model involves not only the internal demand, but also considering the external demand [1]. Based on the characteristics of economic at North Sumatera with export total amount calculation, then the work could be applied on the solution of each commodity to have their prediction total number production for internal and external demand.

The main findings of this work are prediction of total production number each commodity in North Sumatera by considering internal and external demands. This total number of each commodity is observed for the government and industry for decided economic policy and planning regarding total number production each com-

modity at North Sumatera. This total value shows number of production of North Sematera for internal and external demand. By this analysis, the province will have allocation budget with economic analysis, then total commodity produced has optimum need with price stabilization control by this approach.

2. Literature Review

An economic analysis as the fundamental analysis for the wide range industry on any nation, a region or a state. This analysis predicts the production process at industry and how the effective of this industry is operate. Economic analysis is used in many situations, such it uses an estimated demand model. Stock market analysts use economic models to forecast the profits of companies in order to predict the price of their stocks. For economic process of analysis, it has the solution to divide it into sectors included at the industry. Moreover, the analysis also studies how the sectors interact with one another as the system at economic production [2].

The wide range sectors having the roles at their production and consumption step on their economic systems. This real interact to others systems shows the economic linkage that one production sector needs the other consumption from other sectors. The interaction of one sector to others on the economic system has producing some output and will require the input from other sectors and itself. This Therefore the economic process is the network input and output flow in and out of the sector [3]. Therefore, this process includes input output process at the economic system need input output analysis.

The evaluation on input output process related to the open sector, the sectors that do not produce outputs such consumer. This system as the big linkage economic at much sector produces output. But may exist sector that may consume output from other sectors but does not produce output. This study is focused with focus on output levels that are required for the productive sectors to sustain themselves and satisfy the demand of the open sector [4]. The case on input-output model with open sector is constructed by Leontief Model with application such [5] did.

Input-output model of economic flow between sectors applied at some rules at economic analysis such [6]. This study shows that input output model is proposed for analyzing the potential economic benefits at Cipularang Tollroad Investment to Bandung District. By evaluating nine sectors of industry Cipularang Tollroad, this paper depicts that The Industrial sector has the largest forward linkage value and with largest shares on GDP of Bandung district. Therefore, the stakeholders involved in the development of Indonesian transportation infrastructure, by highlighting the broader economic benefits received by the regions.

The previous study at Leontief Model also evaluated to attempt to determine the importance of small seaport at ZPM Ko lobrzeg by using estimation on Leontief Model [7]. This article shows that determined by the multiplier value, which is lower than in previous studies due to the reference area being narrowed down, and by the use of the FLQ formula reducing the self-supplying ability of the Koszalin subregion. The induced and direct effects had the most significant impact on the economy of the subregion, with the indirect effect being of a minor significance.

The Leontief matrix is also present the prediction number of infestation result on the global output at Romania [8]. This study characterized 10 sector groups at 2007 with 105 branches of Rumania's economic model. This model shown at Leontief matrix with the impact of Gross Fixed Capital Formation (GFCF) upon the output having ±5 implication at this case. This forecasting shows the influence of the sectoral structure of the indicator in question is quantified with the help of three different macroeconomic simulations. This result declare the global economic model of the development sector at the system global economic of infestation decisions.

Moreover, the study on Leontief model and the Ghosh model based on the hypothetical extraction method also constructed on the Australian construction industry [9]. This paper indicates that the construction effects have become weaker in relation to promoting economic growth, although the mining and quarrying industry and some service industry activities demonstrate strong and positive backward and forward linkages with the construction industry in Australia. This research provides an approach for measuring corresponding changes in other industries caused by one industry's inter industrial changes.

Basic study on Leontief input-output has the development according to some application needs. Then, this fundamental work has roles on the minimization linear programming on economic system on demand number of quantities. This construction of demand on economic system solve the internal and external need to have the optimum solution on linear programming included at the system. The application of this model also developed by [10] on the inventory control and simulated on six-sector illustration for the utility of the proposed model.

Taking part of some sectors at economic system resulting the effect on demand total number on the system. By using the static Leontief model on Turkish economic system, in [11] shows that the analysis of 2002 Domestic I-O Table by Static L-Model, in Turkey, research and development sector of High Technology Sectors category, manufacture of pulp, paper and paper products of Ricardo Sectors category and recycling, manufacture of basic metals and electricity, gas, steam and hot water supply; collection, purification and distribution of water sectors of Heckscher-Ohlin Sectors category are determined as the strong sectors which may have the highest priority at investment policies of the economy of 52 total sector evaluated.

As the case study of Leontief input-output model, the fresh milk industry construct the linkage distribution system. The model applied on the fresh milk industry on Jabung-Malang Indonesia for evaluating number of distribution linkage of some included sectors. In 2016, the data analysis shows that the merging of the collectors' and the fresh milk processing industry's activities shows high indices of forward linkages and backward linkages. It is shown that merging of the two activities is the key sector which has an important role in developing the whole activities in the fresh milk distribution [12].

3. Theory

3.1. *Leontief Economic Model*

Most sectors of an economy will produce outputs, but there may exist sectors that consume outputs without producing anything themselves (the consumer market, for example). Those sectors that do not produce outputs are called open sectors. Economies with no open sectors are called closed economies, and economies with one or more open sectors are called open economies. In this section we will be concerned with economies with one open sector, and our primary goal will be to determine the output levels that are required for the productive sectors to sustain themselves and satisfy the demand of the open sector [13].

The Leontief system states that all the output of a given industry which is not consumed by other industries is consumed by households. As output in this case is measured by net sales, there can be no doubt as to the veracity of this assertion. This statement may be summarized in the following steps.

Let C is called the consumption matrix (or sometimes the consumption matrix) for the economy. The matrix represent the total number of production inputs required by the every sector included at economic model [14].

Continuing with the next variable at economic Leontief model, let the column vector $\vec{d}$ that has these numbers as successive components is called the outside demand vector. Since the product-producing sectors consume some of their own output, the number of their output must cover their own needs plus the outside demand. Suppose that the dollar values required to do this is the column vector $\vec{x}$ that has these numbers as successive components is called the production vector for the economy.

The vector $C\vec{x}$ is called the intermediate demand vector for the economy. Once the intermediate demand is met, the portion of the production that is left to satisfy the outside demand is $\vec{x} - C\vec{x}$. Thus, if the outside demand vector is $\vec{d}$, then $\vec{x}$ must satisfy the equation $\vec{x} - C\vec{x} = \vec{d}$ which we will find convenient to rewrite as

$$(I - C)\vec{x} = \vec{d}. \tag{3.1}$$

The matrix $I - C$ is called the Leontief matrix and (3.1) is called the Leontief equation.

3.2. *Linear Algebra*

The $\vec{x}$ is the production vector needed to fill both the internal needs and the external demand. We start with $\vec{d} = \vec{x} - C\vec{x}$. This means that our demand is equal to our total production minus the production needed by other industries as inputs, where total production $\vec{x}$ is the cumulative product made by each industry whether it is used in production or not. The production needed by other industries as inputs $A\vec{x}$ is the total amount of product that is used in production [15].

When making projections for the future you are not given the total production needed. The relations between industries, the technology matrix A, is known and so is the demand for each industry $\vec{d}$. Our goal would be to find the total production

that will be needed to fill a certain demand. We must solve the equation $\vec{d} = \vec{x} - C\vec{x}$ for $\vec{x}$.

Our initial equation is:

$$\vec{d} = \vec{x} - C\vec{x}. \tag{3.2}$$

Any matrix multiplied by an identity matrix is equal to itself $I\vec{x} = \vec{x}$. Therefore we can replace $\vec{x}$ with $I\vec{x}$,

$$\vec{d} = I\vec{x} - C\vec{x}. \tag{3.3}$$

We factor out an $\vec{x}$ from both terms on the right side of the equation. It is important to factor out the $\vec{x}$ to the right because if it's factored out to the left matrix multiplication will break down when multiplying the demand vector $\vec{d}$ on the left side by $(I - C)^{-1}$:

$$\vec{d} = (I - C)\vec{x}. \tag{3.4}$$

In order to solve for $\vec{x}$ we multiply by $(I - C)^{-1}$ on the left side of both sides of the equation.

$$(I - C)^{-1}\vec{d} = (I - C)^{-1}(I - C)\vec{x}. \tag{3.5}$$

Any matrix multiplied by it's inverse is equal to the identity matrix $(I - A)^{-1}(I - A) = I$. Substituting I for $(I - C)^{-1}(I - C)$ we get

$$(I - C)^{-1}\vec{d} = I\vec{x}. \tag{3.6}$$

Since $I\vec{x} = \vec{x}$ as stated before we substitute X for $I\vec{x}$, $(I - C)^{-1}\vec{d} = \vec{x}$ with a little rearranging we have our equation to solve for the total production needed to satisfy an economy with a known demand vector $\vec{d}$ and a known technology matrix C:

$$\vec{x} = (I - C)^{-1}\vec{d}. \tag{3.7}$$

Let B represent a square matrix and let x and y be column vectors. The equation $Bx = y$ represents a Leontief system when the following conditions are met:

$$b_{ij} \leq 0, i \neq 0, y_i \geq 0, \tag{3.8}$$

$$\sum_{i=}^{n} b_{ij} \leq 0. \tag{3.9}$$

The following theorem as the main condition boundary for Leontief model using linear algebra operation.

Lemma 3.1. [2] *If $Bx \geq 0$ implies $x \geq 0$, then B is nonsingular, where B is any matrix.*

Proof. Let $Bx = 0$ implies $B(-x) = 0$. Then by the hypothesis, it must be true that $x = 0$, and B is nonsingular.

We may now state a lemma which leads directly to the well-known necessary and sufficient conditions for a unique nonnegative solution. Let us refer to a matrix B as positive if $Bx = 0$ implies $x > 0$. Let B now define a Leontief system. □

Lemma 3.2. *[2] A necessary and sufficient condition for B to be positive is that every principal minor of B have at least one column sum greater than zero.*

Proof. Suppose the condition is met, but for some y with $y_i \geq 0$, $Bx = y$, and $x_i \leq 0$ for some i. By identical rearrangements of rows and columns $Bx = y$ may be written:

$$\begin{pmatrix} B_1 & B_2 \\ B_3 & B_4 \end{pmatrix} \begin{pmatrix} x^1 \\ x^2 = \end{pmatrix} \begin{pmatrix} y^1 \\ y^2 \end{pmatrix}, \tag{3.10}$$

where $x^1 < 0$ and $x^2 \geq 0$. This gives $B_1 x^1 + B_2 x^2 = y^1$. Since off-diagonal elements are not positive, $B_2 x^2 \leq 0$. Therefore, $B_1 x^1 \geq 0$. By the hypothesis, $p'B_1 \geq 0$ where $p' = (1, \cdots, 1)$. Thus, $p'B_1 x^1 < 0$, since $x^1 < 0$. But this implies $B_1 x^1$ not ≥ 0. Since we have reached a contradiction, $x^1 < 0$ is excluded and B is positive.

On the other hand, suppose that B is positive but there is a principal minor whose column sums are all equal to zero. Without loss of generality we may suppose that this minor B_1 lies in the upper left-hand corner of B. Since the column sums of B are not negative, the elements in the columns below B_1 must be zero. We may write:

$$B = \begin{pmatrix} B_1 & B_2 \\ 0 & B_4 \end{pmatrix}. \tag{3.11}$$

Since $p'B_1 = 0$, B_1 is singular. Thus there is a nonzero vector x^1 with $B_1 x^1 = 0$. Consider Bx, where $x = (x^1 0)$. Since $Bx = 0$, B is singular, and by Lemma 3.2 B is not positive. $\qquad\square$

We may derive other useful necessary and sufficient conditions very easily from those of Lemma 3.2.

Theorem 3.3. *[2] Let B satisfy conditions (3.8) and (3.9). The following are equivalent.*

(1) Any Leontief system defined by B has a unique nonnegative solution.
(2) Each principal minor of B has at least one column sum positive.
(3) B is nonsingular.
(4) If B is completely decomposed, each indecomposable minor on the diagonal has a positive column sum.
(5) It is possible to perform identical permutations of rows and columns of B so that $\sum_{i=}^{n} b_{ij} \leq 0$ for all j.

The above theorem shows Leontief system needs the main its boundary for the next calculation on linear algebra steps. This condition need to be fulfilled at the first step as the necessary condition then the optimum solution of the calculation is founded. All above lemmas and theorem are taking role on fundamental theory for constructing Leontief process at result and discussion section.

4. Result and Discussion

Total commodity of this study is classified based on Standard International Trade Classification (SITC) on mentioned year. SITC product classification system developed in 1962 by the United Nations (UN) such detailed on Table 1 SITC was created with the aim of classifying the products traded not only based on the material and physical properties of the product but, also in accordance with the processing stage as well as the economic function of the product in order to facilitate economic analysis.

Table 1. Standard International Trade Classification (SITC)

STIC Code	Commodity
0	Food & Live Animals
1	Beverages & Tobacco
2	Raw Material
3	Fuels & Lubricants
4	Animal & Vegetable Oils
5	Chemicals
6	Manufactured Goods
7	Machinery & Transport Equipment
8	Miscellaneous Manufactures
9	Others

The following data depicts the total export number on last five years on North Sumatera by the SITC classification mentioned on Table 2. This trend shows that on the 2021 it has increasingly total export compared with two years before on pandemic demand. The data informs that the recent year on North Sumatera has positive economic total export after pandemic situation. Focusing on the evaluation total commodity on North Sumatera on Million US$ by considering export need, as the main input for export total number at most recent year, 2021 as the fundamental information.

Evaluating total demand of those commodity of North Sumatera, the total number export is delivered on some connected countries. The main countries having relation with North Sumatera is specialized on 4-contry export, such United State, China, India and Japan. Those countries as the counties of destination on export trade of North Sumatera. The total export each country on 2021 from North Sumatera in Billion US$ is described such Table 3.

In Table 3, as the system included on North Sumatera export demand. By using the next discussion, all the number export related to the North Sumatera trade also mentioned before. Therefore, each county has the total export number of data for this study purpose. After getting mentioned data, our main fundamental information on total demand of each country. This total demand as the number

Table 2. Total Export North Sumatera based on SITC

SITC 1 Digit	Export based on SITC 1 Digit FOB Number (Million US$)				
	2017	2018	2019	2020	2021
Food & Live Animals	1519.81	1547.23	1465.71	1508.48	1971.60
Beverages & Tobacco	323.03	293.96	303.41	253.64	205.76
Raw Material	1391.65	1084.47	915.46	751.41	958.97
Fuels & Lubricants	0.02	0.07	0.27	0.13	0.45
Animal & Vegetable Oils	3396.56	136.47	2622.78	2860.61	4729.69
Chemicals	1730.46	1794.62	1514.96	1596.80	2509.58
Manufactured Goods	389.00	451.56	350.13	40.25	556.92
Machinery & Transport Equipment	139.14	132.94	158.88	152.38	163.35
Miscellaneous Manufactures	332.93	343.25	344.83	522.54	778.36
Others	2.71	2.66	2.13	0	0
Total	**9 225.29**	**8787.22**	**7678.56**	**8086.22**	**11874.67**

Table 3. Total Export 2021 (US$ Billion)

Country	North Sumatera	United State	China	India	Japan
North Sumatera	206.58	0.56	1.74	0.77	1.48
United State	0.34	22996.10	151.07	40.13	74.96
China	1.51	577.13	14370.29	97.51	165.82
India	0.24	71.51	23.04	2782.43	6.08
Japan	0.07	135.98	163.86	12.86	4595.19

of commodity need with considering total number consumption export number all countries. The data as the total commodity export on 2021 of each country to the all demand on Billion US$, is declared on Table 4.

Table 4. Total Demand 2021 (US$ Billion)

Country	Total Demand
North Sumatera	231.61
United State	2556,64
China	3363,77
India	390,97
Japan	604,81

Studying related to the main goal on model recommendation on export number of North Sumatera, it is evaluated using Leontief model theory using algebra operation. The solution of the Leontief model is solved manually and also simulated by using Maple software as the tool for this problems easier. The process of the study is organized as the following result solutions on concerning Leontief model economy, this output is continued to the next step interpretation on discussion steps.

The discussion main on Leontief model economy, first we have to construct the consumption matrix for the economy. The data of this consumption matrix is constructed based on the data on Table 3 representing inputs required by each country to produce output of total export with internal and external needs. Therefore, the matrix consumption export written as:

$$C = \begin{pmatrix} 0.4920009431 & 0.000023341 & 0.000116613 & 0.000244846 & 0.000292799 \\ 0.000014486 & 0.488822351 & 0.00397895 & 0.001531896 & 0.002667034 \\ 0.000097918 & 0.014800657 & 0.480254164 & 0.005373648 & 0.008267937 \\ 0.000077619 & 0.002681835 & 0.001309589 & 0.478327316 & 0.000786869 \\ 0.000013681 & 0.004739756 & 0.008352550 & 0.001639959 & 0.471229525 \end{pmatrix}, \quad (4.1)$$

with the inverse of this matrix can be written as the following form:

$$C^{-1} = \begin{pmatrix} 2.032517 & -0.000065 & -0.000468 & -0.001031 & -0.001253 \\ -0.000056 & 2.046384 & -0.016741 & -0.006327 & -0.011278 \\ -0.000408 & -0.062606 & 2.083441 & -0.023081 & -0.036162 \\ -0.000328 & -0.011270 & -0.005550 & 2.090729 & -0.003330 \\ -0.000050 & -0.019434 & -0.036741 & -0.006803 & 2.122874 \end{pmatrix}. \quad (4.2)$$

By the above result, we can find the $(I - C)$ form by evaluating inverse at the first step on the C matrix.

The column vector $\vec{x}$ that has these numbers as successive components is called the production vector for the economy. For the economy with consumption (4.2), that portion of the production vector $\vec{x}$ that will be consumed by the five productive country is written as:

$$\begin{pmatrix} \text{North Sumatera} \\ \text{United State} \\ \text{China} \\ \text{India} \\ \text{Japan} \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix}. \quad (4.3)$$

Moreover, based on total demand mentioned data before, then the data represents as outside demand vector of Leontief process. The vector constructs of total demand each country including North Sumatera of total needs is written as:

$$d = \begin{pmatrix} 231.61 \\ 2556.64 \\ 3363.77 \\ 390.97 \\ 604.81 \end{pmatrix}. \quad (4.4)$$

After getting all Leontief parameters, this stage we construct Leontief matrix by applying identity matrix and operate with matrix consumption export [16], it results:

$$I-C = \begin{pmatrix} 0.507990569 & -0.000023341 & -0.000116613 & -0.000244846 & -0.000292799 \\ -0.000014486 & 0.511177649 & -0.00397895 & -0.001531896 & -0.002667034 \\ -0.000097918 & -0.014800657 & 0.519745835 & -0.005373648 & -0.008267937 \\ -0.000077619 & -0.002681835 & -0.001309589 & 0.521672684 & -0.000786869 \\ -0.000013681 & -0.004739756 & -0.008352550 & -0.001639959 & 0.528770475 \end{pmatrix}.$$

(4.5)

The above $(I - C)$ operation having the entries of matrix representing Leontief matrix. Then this form of matrix is used by Leontief equation (3.1) based on the Lemma 3.1, 3.2, and Theorem 3.3 with considering five countries vector x_1, x_2, x_3, x_4, x_5 and also total demand vector each country for the system operation. These relation forms Leontief basic form such:

$$\begin{pmatrix} 0.507990569 & -0.000023341 & -0.000116613 & -0.000244846 & -0.000292799 \\ -0.000014486 & 0.511177649 & -0.00397895 & -0.001531896 & -0.002667034 \\ -0.000097918 & -0.014800657 & 0.519745835 & -0.005373648 & -0.008267937 \\ -0.000077619 & -0.002681835 & -0.001309589 & 0.521672684 & -0.000786869 \\ -0.000013681 & -0.004739756 & -0.008352550 & -0.001639959 & 0.528770475 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} 231.61 \\ 2556.64 \\ 3363.77 \\ 390.97 \\ 604.81 \end{pmatrix}.$$

(4.6)

The process of Gauss Jordan Elimination or row echelon step by step is calculated by using Maple application, then the solution of these system resulted:

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} 458.8218267 \\ 5062.3456860 \\ 6645.0345550 \\ 749.1848187 \\ 1296.6230280 \end{pmatrix}.$$

(4.7)

Final step of the process shows that five countries vector x_1, x_2, x_3, x_4, x_5 has the number solution 458.8218267, 5062.3456860, 6645.0345550, 794.1848187, 1296.6230280 respectively. This final result having total optimum with each variable with input of the system on Leontief model [16].

Based on the result of Leontief process before, the operation having the final value where each variable having representation such described on Table 5.

Table 5. Total Production Each Country (US$ Billion)

Country	Total Production (US$ Billion)
North Sumatera	458.8218267
United State	5062.3456860
China	6645.0345550
India	794.1848187
Japan	1296.6230280

Total production of North Sumatera, United State, China, India, Japan is 458.8218267, 5062.3456860, 6645.0345550, 794.1848187, 1296.6230280 US$ Billion respectively. These total number are the amount of commodity that have to produced each country for export purposes of other countries. For North Sumatera results, to deal with export needs of United State, China, India and Japan, North Sumatera have to produce 458.8218267 US$ Billion commodity on the export system on Leontief model.

The prediction number of total exports of North Sumatera presents the number of total productions to fulfill the internal and external demand such North Sumatera and other countries number. From the number of total production export at North Sumatera having prediction point, it means that production control of export commodity has stability demand trade. Focusing on price stability of all commodities on export at North Sumatera by controlling total commodity to the next total production not only internal but also for external number demand.

This model of solution using Leontief model also can be applied on calculation on total production of North Sumatera regarding to the kinds of commodity, involved sectors, production chain and other scope based on the system of North Sumatera economic systems. This works will have the main solution for economic stability after pandemic condition to restore economic goals.

5. Conclusion

Economic system has involved some criteria involved at North Sumatera export system. This condition can be solved by using Leontief model economic with input output model that is reliable on the North Sumatera has. This system presents total production number that have to produce for answering total demand of internal and also external demand. The number 458.8218267 US$ Billion as the total production by the step of Leontief model that have to produce to fulfill United State, China, India, Japan and internal demand. This number as the consideration number to the government to take economic policy regarding export number.

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