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# MIXTILINEAR EXCIRCLE ON THE ANGLE TRISECTOR OF A TRIANGLE

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**Abstract**. *The most famous theorem on the angle trisector of a triangle is Morley's theorem, namely the existence of an equilateral triangle formed from the trisector of its three angles. In this article, the mixtilinear excircle concept will be developed for triangles formed by constructing angle trisectors in a triangle. The results obtained will be expressed in the form of a theorem which states the ratio of several mixtilinear excircles whose value is equal to one. Apart from that, the ratio of several mixtilinear excircles can also be expressed in tangen form of the angles formed.*

*Keywords*: Angle trisector, mixtilinear excircle, triangle

## 1. Introduction

Based on the textbooks in [1,2], a method has been provided for calculating the length of an angle bisector in a triangle. One of the topics discussed in [3] on plane geometry is the angle trisector of a triangle, which is a line that divides a triangle's angle into three equal parts. In [4,5], the discussion includes the length of the angle trisector sides in a triangle and the area ratio of the angle trisectors formed from each angle of the triangle. The most famous theorem concerning triangles that serves as an example is [6,7,8] Morley's theorem.

Mixtilinear was first developed by [9] regarding the basic concept of mixtilinear in a specific context. In [10], mixtilinear is divided into two, namely mixtilinear incircle and mixtilinear excircle. for mixtilinear incircle has been developed by [11]. There has been extensive development on mixtilinear and its applications in various geometry problems. The comparison between mixtilinear excircle and mixtilinear incircle was discussed by [12]. In [13], further developments on the mixtilinear excircle were made. The discussion in [14] concerns the relationship between the radius of the mixtilinear circle and the Finsler–Hadwiger inequality in triangle geometry.

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The use of symbolic geometry is discussed in [15] to prove theorems related to the radii of the mixtilinear excircle and incircle. Recently, mixtilinear incircles and excircles on the angle bisectors of triangles have been developed. Previously, the topic of mixtilinear excircle on the angle trisectors of a triangle had not been discussed. In this work, several new theorems will be obtained as an extension of the mixtilinear excircle concept previously found in triangles.

## 2. Some Concepts

The following are some definitions and theorems that will support this discussion.

### 2.1. *The Circumcircle of a Triangle*

In any triangle $ABC$, [1,2] defines the circumcenter (the center of the circumcircle) as the intersection point of the three lines that pass through the midpoints and are perpendicular to each side of the triangle.

**Definition 2.1.** [1, 2] *The circumcircle of a triangle $ABC$ is the circle that passes through all three vertices of the triangle, and its center is called the circumcenter.*

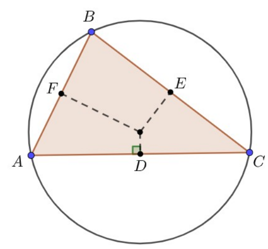


Figure. 1. Circumcircle

In the circumcircle, the relationship between the radius and the sides of the triangle can be expressed using the sine rule, as stated in the following theorem.

**Theorem 2.2.** [1, 2] *Let the side lengths of $\triangle ABC$ be $a, b,$ and $c$, and let $R$ be the radius of the circumcircle of $\triangle ABC$. Then the following relationship holds:*

$$\frac{a}{\sin \angle A} = \frac{b}{\sin \angle B} = \frac{c}{\sin \angle C} = 2R. \tag{2.1}$$

The following is the relationship between the radius of an outer circle and the product of the lengths of its three sides divided by a multiple of the area of the area circle.

**Theorem 2.3.** [1, 2] *If $a, b,$ and $c$ are the side lengths of $\triangle ABC$ and $L$ is the area of the triangle, then the following holds:*

$$R = \frac{abc}{4L}. \tag{2.2}$$

### 2.2. *The Excircle of a Triangle*

According to [7], the excircle is defined as follows.

**Definition 2.4.** [*1, 2*] *An excircle of a $\triangle ABC$ is a circle that is tangent to one side of the triangle and the extensions of the other two sides.*

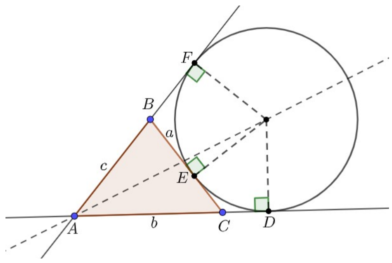


Figure. 2. Excircle of a Triangle

Similar to the problems concerning the incircle and excircle by [10], the radius of the excircle of a triangle can be determined using the following theorem.

**Theorem 2.5.** [*1, 2*] *Let $\triangle ABC$ be an arbitrary triangle, and let $R_a$ be the radius of the excircle opposite to $\angle BAC$ with respect to side $BC$. Then the radius of the excircle opposite side $BC$ is given by:*

$$R_a = s \tan \frac{1}{2} \angle A. \tag{2.3}$$

Besides using the above equation based on [10], the formula for finding the radius of the excircle of a triangle can be expressed as follows.

**Theorem 2.6.** [*1, 2*] *Let $\triangle ABC$ be an arbitrary triangle, and let $L$ be the area of $\triangle ABC$. Then, the radius of the excircle opposite side $BC$ is:*

$$R_a = \frac{L}{s - a}. \tag{2.4}$$

### 2.3. *The Angle Trisectors of a Triangle*

Besides dividing an angle into two equal parts in a triangle, according to [4], if two lines are drawn from each vertex to the opposite side, the angle can be divided into three equal parts.

**Definition 2.7.** [*4*] *The angle trisectors of a triangle consist of two dividing lines that split the angle into three equal parts.*

In Figure 3, $AA_1$ and $AA_2$ are trisector lines drawn from $\angle A$, dividing it into three equal angles. Trisectors are often discussed in a theorem known as Morley's theorem. According to [4], Morley's theorem is one of the most fascinating aspects of twentieth-century geometry. The following theorem applies to finding the lengths of the trisector lines.

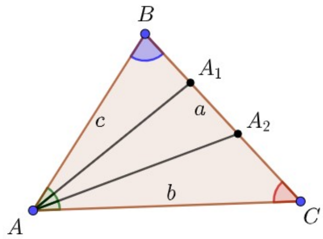


Figure. 3. Angle Trisectors of a Triangle

**Theorem 2.8.** [4] *In $\triangle ABC$, if angle trisectors are drawn, there are two trisector lines, $AA_1$ and $AA_2$, which divide $\angle A$ into three equal parts. With side lengths $BC = a, AC = b, AB = c$ and $\angle A = \alpha, \angle B = \beta$ and $\angle C = \gamma$, the lengths of $AA_1$ and $AA_2$ are:*

$$AA_1 = \frac{2L}{\alpha \sin(\frac{\alpha}{3} + \beta)}, \tag{2.5}$$

$$AA_2 = \frac{2L}{\alpha \sin(\frac{\alpha}{3} + \gamma)}. \tag{2.6}$$

### 2.4. *Mixtilinear Excircle*

The mixtilinear excircle is defined [10] analogously to the mixtilinear incircle, except that it is tangent externally. The following is the definition of the mixtilinear excircle in a triangle.

**Definition 2.9.** [10] *A mixtilinear excircle is a circle that is tangent to the extensions of two sides of a triangle and to the triangle's circumcircle externally.*

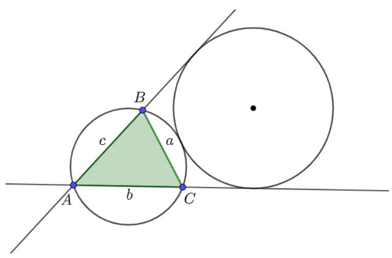


Figure. 4. Mixtilinear Excircle

There have been many developments from [9] regarding mixtilinear circles. One of these is a theorem for finding the radius of the mixtilinear excircle.

**Theorem 2.10.** [13] *Let $a, b$ and $c$ be the side lengths of $\triangle ABC$, and let a mixtilinear excircle be constructed opposite $\angle A$ with radius $\rho_{BAC}$. The circumcircle of*

*$\triangle ABC$ has radius $R$, and $\alpha$ is the measure of $\angle A$. Then, the following holds:*

$$\rho_{BAC} = R_{BAC} \sec^2 \frac{1}{2}\angle A. \tag{2.7}$$

In an arbitrary $\triangle ABC$, two lines are drawn to divide $\angle BAC$ into three equal parts, creating $\triangle EAC, \triangle EAD, \triangle DAB$ and $\triangle ABC$. Three different mixtilinear excircles are formed in each of these triangles. Thus, the radius of the mixtilinear excircle obtained can be found from the following theorem.

**Theorem 2.11.** *Let $a, b$, and $c$ be the lengths of the sides of triangle $ABC$, and let the mixtilinear excircle with respect to angle $A$ be constructed, with radius $\rho_{BAC}$. Let $\alpha$ denote the measure of angle $A$. Then, the following holds*

$$\rho_{BAC} = \frac{2bc}{(b+c-a)} \tan \frac{1}{2}\alpha. \tag{2.8}$$

## 3. Result and Discussion

This section presents developments from the concept of the mixtilinear excircle applied to the trisectors of a triangle's angles. By constructing a mixtilinear excircle on each angle trisector, multiple mixtilinear excircles are formed. A comparison of the radii of these mixtilinear excircles then reveals a unique result for a given triangle.

### 3.1. *The Radius of the Mixtilinear Excircle on the Angle Trisector of a Triangle*

In an arbitrary $\triangle ABC$, two lines are drawn to divide $\angle BAC$ into three equal parts, creating $\triangle EAC, \triangle EAD, \triangle DAB$ and $\triangle ABC$. Three different mixtilinear excircles are formed in each of these triangles. Thus, the radius of the mixtilinear excircle obtained can be found from the following theorem.

**Theorem 3.1.** *Let $\triangle ABC$ be an angle trisector with side lengths $BC = a, AC = b, AB = c$, and the lengths of the trisector lines from $\angle A$ are $AE = d$ and $AD = e$. In any $\triangle ABC$, two line are drawn to divide the $\angle BAC$ into three equal parts, forming $\triangle CAE, \triangle EAD, \triangle DAB$ and $\triangle ABC$. Three distinct mixtilinear excircles are constructed for each of these triangles, resulting in the following theorem for the radius lengths of the mixtilinear excircles:*

$$\rho_{CAE} = \frac{2bf}{(b+d-f)} \tan \frac{1}{2}\alpha, \tag{3.1}$$

$$\rho_{AEC} = \frac{2df}{(d+f-b)} \tan \frac{1}{2}\theta, \tag{3.2}$$

$$\rho_{ACE} = \frac{2bf}{(b+f-d)} \tan \frac{1}{2}\gamma. \tag{3.3}$$

**Proof.** Using Theorem 2.10, the radius of the mixtilinear excircle will be derived

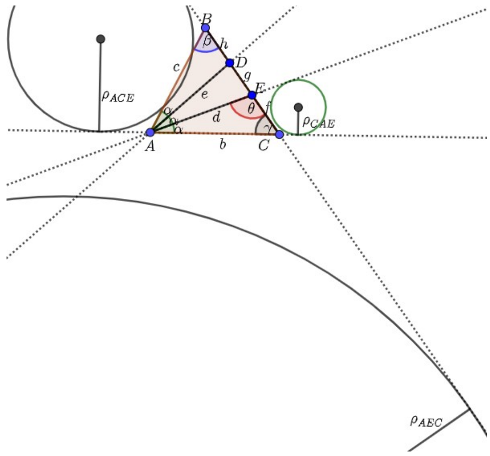


Figure. 5. Illustration of Theorem 3.1

by applying Theorem 2.6.

$$\rho_{CAE} = R_{CAE} \sec^2 \frac{1}{2}\alpha = \frac{L\triangle CAE}{(s_{CAE} - f)\sec^2 \frac{1}{2}\alpha}.$$

And by using Theorem 2.3, the following is obtained:

$$\rho_{CAE} = \frac{\frac{bdf}{4R'_{CAE}}}{(s_{CAE} - f)} \sec^2 \frac{1}{2}\alpha = \frac{bdf}{4R'_{CAE}(s_{CAE} - f)} \sec^2 \frac{1}{2}\alpha.$$

Since $s_{CAE}$ is half of the perimeter of $\triangle CAE$, it follows that:

$$\rho_{CAE} = \frac{bdf}{4R'_{CAE}(\frac{1}{2}(b+d+f) - f)} \sec^2 \frac{1}{2}\alpha = \frac{bdf}{4R'_{CAE}(b+d-f)} \sec^2 \frac{1}{2}\alpha.$$

Using Theorem 2.2, it is obtained that:

$$\rho_{CAE} = \frac{bdf}{(b+d-f)} \frac{\sin\alpha}{f} \sec^2 \frac{1}{2}\alpha = \frac{bd}{(b+d-f)} \sin\alpha \sec^2 \frac{1}{2}\alpha.$$

Using the obtained trigonometry rules:

$$\frac{bd}{(b+d-f)} 2\sin\frac{1}{2}\alpha\cos\frac{1}{2}\alpha \frac{1}{\cos^2 \frac{1}{2}\alpha} = \frac{2bd}{(b+d-f)} \tan\frac{1}{2}\alpha.$$

Thus,

$$\rho_{CAE} = \frac{2bd}{(b+d-f)} \tan\frac{1}{2}\alpha.$$

Thus, in a similar manner, we obtain

$$\rho_{AEC} = \frac{2df}{(d+f-b)} \tan\frac{1}{2}\theta,$$

$$\rho_{ACE} = \frac{2bf}{(b+f-d)} \tan\frac{1}{2}\gamma.$$

□

### 3.2. *Mixtilinear Excircle on the Angle Trisectors of a Triangle*

The following theorem presents the result obtained after constructing a mixtilinear excircle on the trisectors of a triangle's angles.

**Theorem 3.2.** *Let the angle trisectors of* $\triangle ABC$ *at* $\angle A$ *have side lengths* $BC = a, AC = b$ *and* $AB = c$*. The lengths of the trisector lines at* $\angle A$ *are* $AE = d, AD = e, CE = f, ED = g$ *and* $BD = h$*. Given that* $\angle CAE = \angle EAD = \angle DAB = \alpha, \angle ABC = \angle ABD = \beta$ *and* $\angle BCA = \angle ECA = \gamma$*, the relationship for the radius of the mixtilinear excircle in* $\triangle ABC$ *is obtained as: follows:*

$$\frac{\rho_{CAE}\rho_{ABD}\rho_{BCA}}{\rho_{DAB}\rho_{ABC}\rho_{ECA}} = 1. \tag{3.4}$$

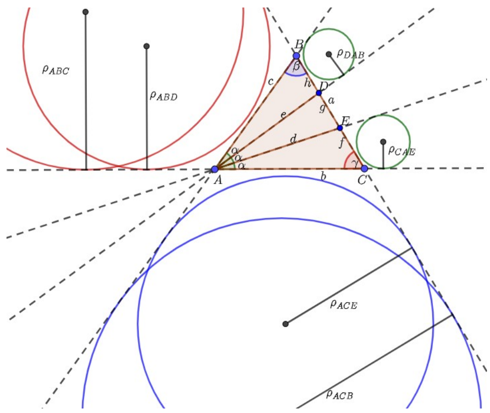

Figure. 6. Illustration of Theorem 3.2

**Proof.** By substituting the radius formula into the Equation 3.4, the following is obtained:

$$\frac{\rho_{CAE}\rho_{ABD}\rho_{BCA}}{\rho_{DAB}\rho_{ABC}\rho_{ECA}} = \frac{\frac{2bd}{(b+d-f)}\tan\frac{1}{2}\angle A\frac{2ch}{(c+h-e)}\tan\frac{1}{2}\angle B\frac{2ab}{(a+b-c)}\tan\frac{1}{2}\angle C}{\frac{2ce}{(c+e-h)}\tan\frac{1}{2}\angle A\frac{2ac}{(a+c-b)}\tan\frac{1}{2}\angle B\frac{2bf}{(b+f-d)}\tan\frac{1}{2}\angle C}$$
$$= \frac{bdh(e+c-h)(c+a-b)(b+f-d)}{ecf(b+d-f)(c+h-e)(a+b-c)}$$

By applying the sine rule and the area ratio of the triangle on the angle trisectors, it can be simplified to:

$$\begin{aligned}
\frac{\rho_{CAE}\rho_{ABD}\rho_{BCA}}{\rho_{DAB}\rho_{ABC}\rho_{ECA}} &= \frac{\sin\beta\sin(\alpha+\beta)\sin(\alpha+\gamma)\sin\gamma(e+c-h)(c+a-b)(b+f-d)}{\sin(\alpha+\gamma)\sin\gamma\sin(\alpha+\beta)\sin\beta(b+d-f)(c+h-e)(a+b-c)},\\
&= \frac{(e+c-h)(c+a-b)(b+f-d)}{(b+d-f)(c+h-e)(a+b-c)},\\
&= \frac{\frac{1}{2}(e+c-h)\frac{1}{2}(c+a-b)\frac{1}{2}(b+f-d)}{\frac{1}{2}(b+d-f)\frac{1}{2}(c+h-e)\frac{1}{2}(a+b-c)},\\
&= \frac{(\frac{1}{2}(e+c+h)-h)(\frac{1}{2}(c+a+b)-b)(\frac{1}{2}(b+f+d)-d)}{(\frac{1}{2}(b+d+f)-f)(\frac{1}{2}(c+h+e)-e)(\frac{1}{2}(a+b+c)-c)}.
\end{aligned}$$

Since $s$ is half of the triangle's perimeter, then:

$$\frac{\rho_{CAE}\rho_{ABD}\rho_{BCA}}{\rho_{DAB}\rho_{ABC}\rho_{ECA}} = \frac{(s_{BAD}-h)(s_{ABC}-b)(s_{ACE}-d)}{(s_{CAE}-f)(s_{ABD}-e)(s_{ACB}-c)}.$$

Once again, by using Theorem 2.6, we have:

$$\frac{\rho_{CAE}\rho_{ABD}\rho_{BCA}}{\rho_{DAB}\rho_{ABC}\rho_{ECA}} = \frac{\frac{L\triangle BAD}{R_{BAD}}\frac{L\triangle ABC}{R_{ABC}}\frac{L\triangle ACE}{R_{ACE}}}{\frac{L\triangle CAE}{R_{CAE}}\frac{L\triangle ABD}{R_{ABD}}\frac{L\triangle ACB}{R_{ACB}}} = \frac{R_{BAD}R_{ABC}R_{ACE}}{R_{CAE}R_{ABD}R_{ACB}}.$$

By using Theorem 2.5, the following is obtained:

$$\frac{\rho_{CAE}\rho_{ABD}\rho_{BCA}}{\rho_{DAB}\rho_{ABC}\rho_{ECA}} = \frac{s_{BAD}tan\frac{1}{2}\beta s_{ABC}tan\frac{1}{2}\gamma s_{ACE}tan\frac{1}{2}\alpha}{s_{CAE}tan\frac{1}{2}\gamma s_{ABD}tan\frac{1}{2}\beta s_{ACB}tan\frac{1}{2}\alpha} = \frac{s_{BAD}s_{ABC}s_{ACE}}{s_{CAE}s_{ABD}s_{ACB}}.$$

Since $s$ is half of the triangle's perimeter, then:

$$\frac{\rho_{CAE}\rho_{ABD}\rho_{BCA}}{\rho_{DAB}\rho_{ABC}\rho_{ECA}} = \frac{\frac{1}{2}(c+h+e)\frac{1}{2}(c+a+b)\frac{1}{2}(b+d+f)}{\frac{1}{2}(b+d+f)\frac{1}{2}(c+h+e)\frac{1}{2}(c+a+b)} = 1.$$

Therefore,

$$\frac{\rho_{CAE}\rho_{ABD}\rho_{BCA}}{\rho_{DAB}\rho_{ABC}\rho_{ECA}} = 1.$$

□

The following theorem compares the ratio of the mixtilinear excircle by constructing the mixtilinear excircle on the angle trisectors of a triangle, resulting in the tangent values of the angles within the triangle.

**Theorem 3.3.** *Let the angle trisectors of $\triangle ABC$ at $\angle A$ have side lengths $BC = a, AC = b$ and $AB = c$. The lengths of the trisector lines at $\angle A$ are $AE = d, AD = e, CE = f, ED = g$ and $BD = h$. Given that $\angle CAE = \angle EAD = \angle DAB = \alpha, \angle ABC = \beta, \angle BCA = \gamma, \angle AEC = \theta$ and $\angle AED = \epsilon$, the relationship among the radii of the four mixtilinear excircles is obtained as follows:*

$$\begin{aligned}
\frac{\rho_{CAE}\rho_{AED}}{\rho_{EAD}\rho_{AEC}} &= \tan^4\frac{1}{2}\angle AED,\\
\frac{\rho_{EAD}\rho_{ADB}}{\rho_{DAB}\rho_{ADE}} &= \tan^4\frac{1}{2}\angle ADB.
\end{aligned}$$

**Proof.** We will show that:

$$\frac{\rho_{CAE}\rho_{AED}}{\rho_{EAD}\rho_{AEC}} = \tan^4\frac{1}{2}\angle AED.$$

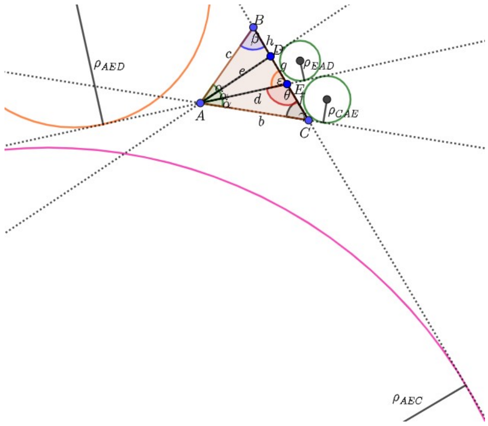


Figure. 7. Illustration of Theorem 3.3

By substituting the radius formula into the equation 3.5, the following is obtained

$$\frac{\rho_{CAE}\rho_{AED}}{\rho_{EAD}\rho_{AEC}} = \frac{\frac{2bd}{b+d-f}\tan(\frac{1}{2}\angle CAE)\frac{2dg}{d+g-e}\tan(\frac{1}{2}\angle AED)}{\frac{2de}{d+e-g}\tan(\frac{1}{2}\angle DAE)\frac{2df}{d+f-b}\tan(\frac{1}{2}\angle AEC)},$$

$$= \frac{bg(d+e-g)(d+f-b)\tan(\frac{1}{2}\angle AED)}{ef(b+d-f)(d+g-e)\tan(\frac{1}{2}\angle AEC)}.$$

By using the sine rule, the following is obtained.

$$\frac{\rho_{CAE}\rho_{AED}}{\rho_{EAD}\rho_{AEC}} = \frac{\sin\angle AEC\sin\angle DAE(d+e-g)(d+f-b)\tan(\frac{1}{2}\angle AED)}{sin\angle AED\sin\angle CAE(b+d-f)(d+g-e)\tan\frac{1}{2}\angle AEC}.$$

Since $\angle AEC$ is related to $\angle AED$, then:

$$\frac{\rho_{CAE}\rho_{AED}}{\rho_{EAD}\rho_{AEC}} = \frac{\sin\angle AEC(d+e-g)(d+f-b)}{\sin\angle AED(b+d-f)(d+g-e)}\tan^2\frac{1}{2}\angle AED.$$

By using the rules of trigonometry we get:

$$\frac{\rho_{CAE}\rho_{AED}}{\rho_{EAD}\rho_{AEC}} = \frac{(d+e-g)(d+f-b)}{(b+d-f)(d+g-e)}\tan^2(\frac{1}{2}\angle AED).$$

The above equation can be expanded as follows:

$$\frac{\rho_{CAE}\rho_{AED}}{\rho_{EAD}\rho_{AEC}} = \frac{(\frac{1}{2}(d+e+g)-g)(\frac{1}{2}(d+f+b)-b)}{(\frac{1}{2}(b+d+f)-f)(\frac{1}{2}(d+g+e)-e)}\tan^2(\frac{1}{2}\angle AED).$$

Since $s$ is half of the triangle's perimeter, then

$$\frac{\rho_{CAE}\rho_{AED}}{\rho_{EAD}\rho_{AEC}} = \frac{(s_{EAD}-g)(s_{AEC}-b)}{(s_{CAE}-f)(s_{AED}-e)}\tan^2(\frac{1}{2}\angle AED).$$

By using Theorem 2.6, the following is obtained:

$$\frac{\rho_{CAE}\rho_{AED}}{\rho_{EAD}\rho_{AEC}} = \frac{\frac{L\triangle EAD}{R_{EAD}}\frac{L\triangle AEC}{R_{AEC}}}{\frac{L\triangle CAE}{R_{CAE}}\frac{L\triangle AED}{R_{AED}}}\tan^2(\frac{1}{2}\angle AED) = \frac{R_{CAE}R_{AED}}{R_{EAD}R_AEC}\tan^2(\frac{1}{2}\angle AED).$$

By using Theorem 2.5, the following is obtained:

$$\frac{\rho_{CAE}\rho_{AED}}{\rho_{EAD}\rho_{AEC}} = \frac{s_{CAE}\tan\frac{1}{2}\angle CAE s_{AED}\tan\frac{1}{2}\angle\angle AED}{s_{EAD}\tan\frac{1}{2}\angle EAD s_{AEC}\tan\frac{1}{2}\angle AEC}\tan^2(\frac{1}{2}\angle AED).$$

Since $s$ is half of the triangle's perimeter, then:

$$\frac{\rho_{CAE}\rho_{AED}}{\rho_{EAD}\rho_{AEC}} = \frac{\frac{1}{2}(b+d+f)\frac{1}{2}(d+g+e)\tan\frac{1}{2}\angle AED}{\frac{1}{2}(d+e+g)\frac{1}{2}(d+f+b)\tan\frac{1}{2}\angle AEC}\tan^2(\frac{1}{2}\angle AED),$$
$$= \frac{\tan\frac{1}{2}\angle AED}{\tan\frac{1}{2}\angle AEC}\tan^2(\frac{1}{2}\angle AED).$$

Since $\angle AEC$ and $\angle AED$ are related, the following is obtained:

$$\frac{\rho_{CAE}\rho_{AED}}{\rho_{EAD}\rho_{AEC}} = \frac{\tan\frac{1}{2}\angle AED}{\frac{1}{\tan\frac{1}{2}\angle AED}}\tan^2(\frac{1}{2}\angle AED).$$

The above equation can be simplified, yielding the following:

$$\frac{\rho_{CAE}\rho_{AED}}{\rho_{EAD}\rho_{AEC}} = \tan^2\frac{1}{2}\angle AED\tan^2\frac{1}{2}\angle AED = \tan^4\frac{1}{2}\angle AED.$$

Thus, it is obtained with the same formula:

$$\frac{\rho_{CAE}\rho_{AED}}{\rho_{EAD}\rho_{AEC}} = \tan^4\frac{1}{2}\angle AED,$$
$$\frac{\rho_{EAD}\rho_{ADB}}{\rho_{DAB}\rho_{ADE}} = \tan^4\frac{1}{2}\angle ADB.$$

□

## 4. Conclusion

Based on the results and discussions presented above, it can be concluded that the concept of the mixtilinear excircle can be developed on the angle trisectors of a triangle. When a mixtilinear excircle is constructed on the triangles formed by the construction of the angle trisectors, and the ratio of various mixtilinear excircles is compared, the result is found to be equal to one, yielding the tangent values of the angles of a triangle formed by the trisected angles.

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