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ON LOCATING-DOMINATING SET OF THE CARTESIAN PRODUCT OF COMPLETE BIPARTITE GRAPHS AND A PATH OF ORDER TWO

SUHADI WIDO SAPUTRO*, IKHSAN RIZQI AZ-ZUKRUF AS-SHIDIQ

Department of Mathematics
Institut Teknologi Bandung
Jl. Ganesa 10 Bandung 40132 Indonesia
email : suhadi@itb.ac.id; ikhsanrizx@gmail.com

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Abstract. *A set D is called as a locating-dominating set of a graph G if D is a dominating set of G such that every vertex outside D is uniquely identified by its neighbourhood within D. The locating-dominating number of G is the minimum cardinality of all locating-dominating sets of G. In this paper, we consider a Cartesian product graphs between a complete bipartite graphs K_{m_1,m_2} and a path of order two P_2, denoted by $K_{m_1,m_2}\Box P_2$. In particular, we determine the locating-dominating number of $K_{m_1,m_2}\Box P_2$ for any values $m_1, m_2 \geq 2$.*

Keywords: Cartesian product, locating-dominating number, locating-dominating set.

1. Introduction

In this paper, we assume that G is a simple and finite graph. Let x be a vertex of G. The *neighbor set* of x in G, denoted by $N(x)$, is the set of vertices which are adjacent to x in G. A set $D \subseteq V(G)$ is called a *locating-dominating* set of a graph G if $\emptyset \neq N(u) \cap D \neq N(v) \cap D \neq \emptyset$ for any two distinct vertices $u, v \in V(G)$. The minimum cardinality of a locating-dominating set of G is called the *locating-dominating number* of G, denoted by $\lambda(G)$. This concept was introduced by Slater [1,2].

It has been proven that the locating-dominating problem is NP-complete [3]. Several authors then tried to construct an efficient algorithm to determine a locating-dominating set for certain classes of graph. Slater [1] formulated a linear-time algorithm for solving locating-dominating set in tree. Meanwhile the precise algorithm for finding the locating-dominating number of graphs which are connected by bridge, can be seen in [4].

*Corresponding author

Although determining the locating-dominating number for any connected graphs is NP-complete, some authors have managed to characterize all graphs with certain locating-dominating number. Henning and Oellermann [5] have proved that the only graphs of order $n \geq 2$ whose locating-dominating number $n-1$ is complete graphs K_n or star graphs $K_{1,n-1}$. They also provided complete characterization of graphs of order $n \geq 4$ with locating-dominating number $n-2$. In [6], Caceres *et al.* have proved that there are 16 non-isomorphic graphs having the locating-dominating number two.

The locating-dominating number of certain class of graphs have been investigated. Henning *et. al.* [5] proved that for $n \geq 3$, the locating-dominating number of complete graphs K_n is $n-1$. The locating-dominating set of some special trees can be seen in [2,7,8]. The locating-dominating number of some regular graphs have been determined in [6,9,10]. Esther and Vivik [11] studied the locating-dominating number of prism, antiprism, crossed prism, and circular ladder prism graph. Meanwhile a locating-dominating set of graphs with girth at least 5 has been investigated in [12].

Determining the locating-dominating number of graphs obtained from a product graphs, is also an interesting problem. The locating-dominating number of corona and composition product graphs have been studied by Canoy Jr. *et. al.* [13]. Pribadi and Saputro [14] determined the locating-dominating number of comb product of any two connected graphs. Argiroffo *et al.* [15] investigated a locating-dominating set of graph by adding a universal vertex.

In this paper, we consider the Cartesian product graphs. The *Cartesian product* graphs between G and H, denoted by $G\Box H$, is a graph with $V(G\Box H) = V(G) \times V(H)$ where two distinct vertices (a,x) and (b,y) are adjacent whenever ($a=b$ and $xy \in E(H)$) or ($ab \in E(G)$ and $x=y$). The study a locating-dominating set in a Cartesian product graphs was initiated by Honkala and Laihonen [16]. In particular, they investigated the problem on grids, which are isomorphic to the Cartesian product between two paths. In 2022, As-Shidiq and Saputro [17] studied the locating-dominating set of $G\Box P_2$ where G is complete graphs, stars, paths, and cycles.

Now, we consider $G\Box P_2$ where G is a complete bipartite graphs K_{m_1,m_2} with $m_1, m_2 \geq 2$. In this paper, we determine the locating-dominating number of $K_{m_1,m_2}\Box P_2$ for any values $m_1, m_2 \geq 2$. In order to do that, we use the notations of $V(K_{m_1,m_2}\Box P_2)$ and $E(K_{m_1,m_2}\Box P_2)$ as stated in Definition 1.1 below.

Definition 1.1. *For* $m_1, m_2 \geq 2$*, the Cartesian product graphs* $K_{m_1,m_2}\Box P_2$ *is a graph with vertex and edge sets as follows.*

$$\begin{aligned} V(K_{m_1,m_2}\Box P_2) &= \{a^*_{i,j}, a^{**}_{i,j} \mid 1 \leq i \leq m_j;\ 1 \leq j \leq 2\} \\ E(K_{m_1,m_2}\Box P_2) &= \{a^*_{i,1}a^*_{j,2}, a^{**}_{i,1}a^{**}_{j,2} \mid 1 \leq i \leq m_1;\ 1 \leq j \leq m_2\} \cup \\ &\quad\ \{a^*_{i,j}a^{**}_{i,j} \mid 1 \leq i \leq m_j;\ 1 \leq j \leq 2\} \end{aligned}$$

An illustration of the graph $K_{m_1,m_2}\Box P_2$ based on definition above, can be seen in Figure 1.

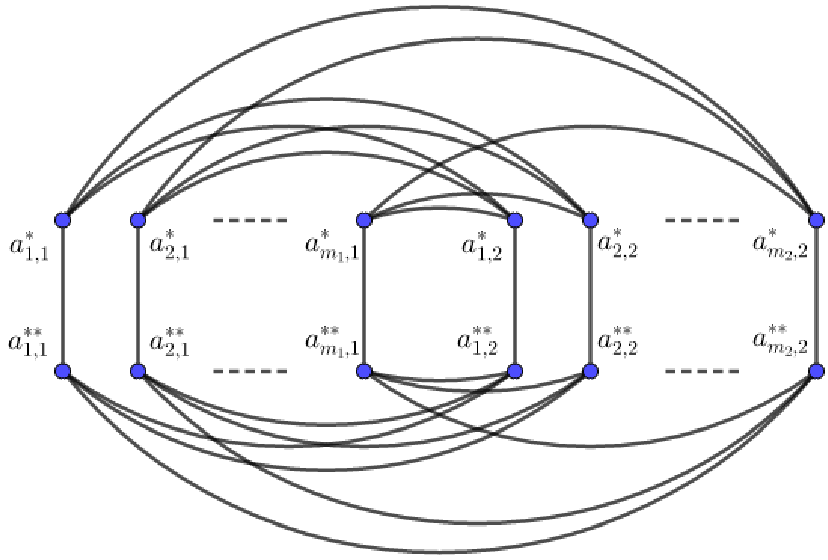

Figure. 1. The graph $K_{m_1,m_2}\Box P_2$ where $m_1, m_2 \geq 2$.

2. Preliminary Results

In this paper, we also use some notations below.

Definition 2.1. *For $1 \leq j \leq 2$ and $1 \leq r \leq m_j$, we define:*

$$\begin{aligned} V_j &= \{a^*_{i,j}, a^{**}_{i,j} \mid 1 \leq i \leq m_j\}, \\ V^*_j &= \{a^*_{i,j} \mid 1 \leq i \leq m_j\}, \\ V^{**}_j &= \{a^{**}_{i,j} \mid 1 \leq i \leq m_j\}, \\ A_{r,j} &= \{a^*_{r,j}, a^{**}_{r,j}\}. \end{aligned}$$

Definition 2.2. *For $m_1, m_2 \geq 2$, let D be a locating-dominating set of $K_{m_1,m_2}\Box P_2$. For $1 \leq j \leq 2$, we define:*

$$\begin{aligned} D_j &= D \cap V_j, \\ D^*_j &= D \cap V^*_j, \\ D^{**}_j &= D \cap V^{**}_j. \end{aligned}$$

Now, let D be a locating-dominating set of $K_{m_1,m_2}\Box P_2$. In proposition below, we show that for $1 \leq j \leq 2$, every two distinct integers $s, t \in \{1, 2, \cdots, m_j\}$ satisfies $D \cap (A_{s,j} \cup A_{t,j}) \neq \emptyset$.

Proposition 2.3. *For $1 \leq j \leq 2$, every two distinct integers $s, t \in \{1, 2, \cdots, m_j\}$ satisfies $D \cap A_{s,j} \neq \emptyset$ or $D \cap A_{t,j} \neq \emptyset$.*

Proof. Suppose that $D \cap A_{s,j} = \emptyset$ and $D \cap A_{t,j} = \emptyset$. Then we obtain $D \cap N(a^*_{s,j}) = D \cap N(a^*_{t,j})$, a contradiction. □

By Proposition 2.3 above, for $1 \le j \le 2$, there is at most one integer $i_j \in \{1, 2, \cdots, m_j\}$ such that $D \cap A_{i_j,j} = \emptyset$. Therefore, we obtain Corollary 2.4 below.

Corollary 2.4. *For $m_1, m_2 \ge 2$, $\lambda(K_{m_1,m_2}\square P_2) \ge m_1 + m_2 - 2$.*

According to Corollary 2.4 above, since $m_1, m_2 \ge 2$, we have $\lambda(K_{m_1,m_2}\square P_2) \ge 2$. However, we will show that $K_{m_1,m_2}\square P_2$ cannot have a locating-dominating set with two or three elements.

In order to do that, let us consider a locating-dominating set D of $K_{m_1,m_2}\square P_2$. By Proposition 2.3, it may be that for $1 \le j \le 2$, there is an integer $s_j \in \{1, 2, \cdots, m_j\}$ such that $D \cap A_{s_j,j} = \emptyset$. In proposition below, we provide a property of such locating-dominating set.

Proposition 2.5. *For $j \in \{1, 2\}$, let D be a locating-dominating set of $K_{m_1,m_2}\square P_2$ where there is an integer $s \in \{1, 2, \cdots, m_j\}$ such that $D \cap A_{s,j} = \emptyset$. Then for $i \in \{1, 2\} \setminus \{j\}$, $D_i^* \ne \emptyset$ and $D_i^{**} \ne \emptyset$.*

Proof. Suppose that $D_i^* = \emptyset$ or $D_i^{**} = \emptyset$. If $D_i^* = \emptyset$, then we have $D \cap N(a_{s,j}^*) = \emptyset$. Otherwise, we have $D \cap N(a_{s,j}^{**}) = \emptyset$. Both conditions follow a contradiction. □

Next, we provide a property of a locating-dominating set D of $K_{m_1,m_2}\square P_2$ where there are integers $s \in \{1, 2, \cdots, m_1\}$ and $t \in \{1, 2, \cdots, m_2\}$ such that $a_{s,1}^*, a_{t,2}^{**} \in D$ but $a_{s,1}^{**}, a_{t,2}^* \notin D$.

Proposition 2.6. *For $j \in \{1, 2\}$, let D be a locating-dominating set of $K_{m_1,m_2}\square P_2$ where there are integers $s \in \{1, 2, \cdots, m_1\}$ and $t \in \{1, 2, \cdots, m_2\}$ such that $a_{s,1}^*, a_{t,2}^{**} \in D$ but $a_{s,1}^{**}, a_{t,2}^* \notin D$. Then there exists $p \in \{1, 2, \cdots, m_1\} \setminus \{s\}$ or $q \in \{1, 2, \cdots, m_2\} \setminus \{t\}$ such that $a_{p,1}^* \in D$ or $a_{q,2}^{**} \in D$.*

Proof. Suppose that every $p \in \{1, 2, \cdots, m_1\} \setminus \{s\}$ and $q \in \{1, 2, \cdots, m_2\} \setminus \{t\}$ satisfy $a_{p,1}^*, a_{q,2}^{**} \notin D$. Then we obtain $D \cap N(a_{s,1}^{**}) = \{a_{s,1}^*, a_{t,2}^{**}\} = D \cap N(a_{t,2}^{**})$, a contradiction. □

Now, we are ready to show that the locating-dominating set of $K_{m_1,m_2}\square P_2$ is at least 4.

Theorem 2.7. *For $m_1, m_2 \ge 2$, $\lambda(K_{m_1,m_2}\square P_2) \ge 4$.*

Proof. Suppose that $\lambda(K_{m_1,m_2}\square P_2) \le 3$. By considering Corollary 2.4, we have $2 \le \lambda(K_{m_1,m_2}\square P_2) \le 3$. We distinguish two cases.

Case 1. $\lambda(K_{m_1,m_2}\square P_2) = 2$.

Let D be a locating-dominating set with $|D| = 2$. Since $m_1, m_2 \ge 2$, by considering Proposition 2.3, $|D_1| = 1 = |D_2|$. Therefore, there exists $i \in \{1, 2, \cdots, m_1\}$, such that $D \cap A_{i,1} = \emptyset$. By Proposition 2.5, then it has to be $D_2^* \ne \emptyset$ and $D_2^{**} \ne \emptyset$. Since $|D_2| = 1$, then we have either $D_2^* = \emptyset$ or $D_2^{**} = \emptyset$, a contradiction.

Case 2. $\lambda(K_{m_1,m_2}\Box P_2) = 3$.

Let D be a locating-dominating set with $|D| = 3$. W.l.o.g, let $|D_1| = 1$ and $|D_2| = 2$. Since $|D_1| = 1$ and $m_1 \geq 2$, there exists $s \in \{1, 2, \cdots, m_1\}$ such that $D \cap A_{s,1} = \emptyset$. By considering Propositions 2.5 and 2.6, we only have one possibility for D_2, that is $D_2 = \{a^*_{p,2}, a^{**}_{p,2}\}$ where $p \in \{1, 2, \cdots, m_2\}$. It follows that there exists $q \in \{1, 2, \cdots, m_2\} \setminus \{p\}$ such that $D_2 \cap A_{q,2} = \emptyset$. By Proposition 2.5, then it has to be $D^*_1 \neq \emptyset$ and $D^{**}_1 \neq \emptyset$. Since $|D_1| = 1$, then we have either $D^*_1 = \emptyset$ or $D^{**}_1 = \emptyset$, a contradiction. □

3. Main Results

In this section, we provide an exact values of the locating-dominating number of $K_{m_1,m_2}\Box P_2$ for any integers $m_1, m_2 \geq 2$. Note that, since K_{m_1,m_2} is isomorphic to K_{m_2,m_1}, without lost of generality, we assume that $2 \leq m_1 \leq m_2$.

Theorem 3.1. *For $m_2 \geq 2$,*

$$\lambda(K_{2,m_2}\Box P_2) = \begin{cases} 4, & \text{if } m_2 \in \{2,3\}, \\ m_2+1, & \text{if } m_2 \geq 4. \end{cases}$$

Proof. First, let $m_2 \in \{2, 3\}$. By Theorem 2.7, it is enough to prove that $\lambda(K_{2,m_2}\Box P_2) \leq 4$. Define $D = \{a^*_{1,1}, a^{**}_{1,1}\} \cup S$ where:

$$S = \begin{cases} \{a^*_{1,2}, a^{**}_{1,2}\}, \text{ if } m_2 = 2, \\ \{a^*_{1,2}, a^{**}_{2,2}\}, \text{ if } m_2 = 3. \end{cases}$$

Note that $|D| = 4$. It is easy to see that D is a locating-dominating set of $K_{2,m_2}\Box P_2$.

Next, we assume that $m_2 \geq 4$. By Corollary 2.4, we have $\lambda(K_{2,m_2}\Box P_2) \geq m_2$. Suppose that D is a locating-dominating set of $K_{2,m_2}\Box P_2$ with m_2 elements. By Proposition 2.3, we have $|D_1| = 1$ and $|D_2| = m_2 - 1$. W.l.o.g, let $D_1 = \{a^*_{1,1}\}$ and $D_2 \cap A_{m_2,2} = \emptyset$. It follows that $D \cap N(a^{**}_{m_2,2}) = \emptyset$, a contradiction. So, we obtain $\lambda(K_{2,m_2}\Box P_2) \geq m_2 + 1$.

Now, we will construct a locating-dominating set of $K_{2,m_2}\Box P_2$ with $m_2 + 1$ elements. Define $W = \{a^*_{1,1}, a^{**}_{1,1}, a^*_{1,2}, a^*_{2,2}\} \cup \{a^{**}_{i,2} \mid 3 \leq i \leq m_2 - 1\}$. Note that $|W| = m_2 + 1$. For every vertex outside W, we obtain the following properties.

$$\begin{aligned} W \cap N(a^*_{2,1}) &= \{a^*_{1,1}, a^*_{1,2}\}, \\ W \cap N(a^{**}_{2,1}) &= \{a^{**}_{i,2} \mid 3 \leq i \leq m_2 - 1\}, \\ W \cap N(a^*_{i,2}) &= \{a^*_{1,1}, a^{**}_{i,2}\};\ (3 \leq i \leq m_2 - 1) \\ W \cap N(a^*_{m_2,2}) &= \{a^*_{1,1}\}, \\ W \cap N(a^{**}_{i,2}) &= \{a^*_{i,2}, a^{**}_{1,1}\};\ (1 \leq i \leq 2) \\ W \cap N(a^{**}_{m_2,2}) &= \{a^{**}_{1,1}\}. \end{aligned}$$

Since every two distinct vertices $u, v \in V(K_{2,m_2}\Box P_2) \setminus W$ satisfies $\emptyset \neq W \cap N(u) \neq W \cap N(v) \neq \emptyset$, we obtain that W is a locating-dominating set of $K_{2,m_2}\Box P_2$. □

Theorem 3.2. *For $m_2 \geq 3$,*

$$\lambda(K_{3,m_2}\Box P_2) = \begin{cases} m_2+2, \text{ if } m_2 \in \{3,4\}, \\ m_2+1, \text{ if } m_2 \geq 5. \end{cases}$$

Proof. We distinguish two cases.

(1) $m_2 \in \{3,4\}$.

Suppose that $\lambda(K_{3,m_2}\Box P_2) \leq m_2+1$. By Corollary 2.4, we have $\lambda(K_{3,m_2}\Box P_2) = m_2+1$. Let D be a locating-dominating set of $\lambda(K_{3,m_2}\Box P_2)$ with m_2+1 elements. By considering Proposition 2.3, WLOG, let $|D_1| = 2$. The only possibility such that every vertex u outside D satisfies $D \cap N(u) \neq \emptyset$ is WLOG $D_1 = \{a^*_{1,1}, a^{**}_{2,1}\}$ and

$$D_2 = \begin{cases} \{a^*_{1,2}, a^{**}_{2,2}\}, & \text{if } m_2 = 3, \\ \{a^*_{1,2}, a^*_{2,2}, a^{**}_{3,2}\}, & \text{if } m_2 = 4. \end{cases}$$

It implies the set $D = D_1 \cup D_2$ is contradicted with Proposition 2.6.

Now, we will construct a locating-dominating set of $K_{3,m_2}\Box P_2$ with m_2+2 elements. Define

$$W = \begin{cases} \{a^*_{1,1}, a^{**}_{2,1}, a^*_{1,2}, a^{**}_{1,2}, a^*_{2,1}\}, & \text{if } m_2 = 3, \\ \{a^*_{1,1}, a^{**}_{2,1}, a^*_{1,2}, a^{**}_{1,2}, a^*_{2,1}, a^*_{3,1}\}, & \text{if } m_2 = 4. \end{cases}$$

Note that $|W| = m_2+2$. It is easy to see that W is a locating-dominating set of $K_{3,m_2}\Box P_2$.

(2) $m_2 \geq 5$.

By Corollary 2.4, it is enough to prove that $\lambda(K_{3,m_2}\Box P_2) \leq m_2+1$. Define $W = \{a^*_{1,1}, a^{**}_{2,1}, a^*_{1,2}, a^*_{2,2}\} \cup \{a^{**}_{i,2} \mid 3 \leq i \leq m_2-1\}$. Note that $|W| = m_2+1$. For every vertex outside W, we obtain the following properties.

$$\begin{aligned} W \cap N(a^*_{2,1}) &= \{a^{**}_{2,1}, a^*_{1,2}, a^*_{2,2}\}, \\ W \cap N(a^*_{3,1}) &= \{a^*_{1,2}, a^*_{2,2}\}, \\ W \cap N(a^{**}_{1,1}) &= \{a^*_{1,1}\} \cup \{a^{**}_{i,2} \mid 3 \leq i \leq m_2-1\}, \\ W \cap N(a^{**}_{3,1}) &= \{a^{**}_{i,2} \mid 3 \leq i \leq m_2-1\}, \\ W \cap N(a^*_{i,2}) &= \{a^*_{1,1}, a^{**}_{i,2}\}; \; (3 \leq i \leq m_2-1), \\ W \cap N(a^*_{m_2,2}) &= \{a^*_{1,1}\}, \\ W \cap N(a^{**}_{i,2}) &= \{a^{**}_{2,1}, a^*_{i,2}\}; \; (1 \leq i \leq 2), \\ W \cap N(a^{**}_{m_2,2}) &= \{a^{**}_{2,1}\}. \end{aligned}$$

Since every two distinct vertices $u, v \in V(K_{3,m_2}\Box P_2) \setminus W$ satisfies $\emptyset \neq W \cap N(u) \neq W \cap N(v) \neq \emptyset$, we obtain that W is a locating-dominating set of $K_{3,m_2}\Box P_2$. $\square$

Theorem 3.3. *For $4 \leq m_1 \leq m_2$, $\lambda(K_{m_1,m_2}\Box P_2) = m_1+m_2-2$.*

Proof. By Corollary 2.4, it is enough to prove that $\lambda(K_{3,m_2}\Box P_2) \leq m_1+m_2-2$. Define $W = \{a^*_{1,1}, a^*_{1,2}, a^*_{2,2}\} \cup \{a^{**}_{i,1} \mid 2 \leq i \leq m_1-1\} \cup \{a^{**}_{i,2} \mid 3 \leq i \leq m_2-1\}$.

Note that $|W| = m_1 + m_2 - 2$. For every vertex outside W, we obtain the following properties.

$$\begin{aligned}
W \cap N(a^*_{i,1}) &= \{a^{**}_{i,1}, a^*_{1,2}, a^*_{2,2}\};\ (2 \le i \le m_1 - 1),\\
W \cap N(a^*_{m_1,1}) &= \{a^*_{1,2}, a^*_{2,2}\},\\
W \cap N(a^{**}_{1,1}) &= \{a^*_{1,1}\} \cup \{a^{**}_{i,2} \mid 3 \le i \le m_2 - 1\},\\
W \cap N(a^{**}_{m_1,1}) &= \{a^{**}_{i,2} \mid 3 \le i \le m_2 - 1\},\\
W \cap N(a^*_{i,2}) &= \{a^*_{1,1}, a^{**}_{i,2}\};\ (3 \le i \le m_2 - 1),\\
W \cap N(a^*_{m_2,2}) &= \{a^*_{1,1}\},\\
W \cap N(a^{**}_{i,2}) &= \{a^*_{i,2}\} \cup \{a^{**}_{j,1} \mid 2 \le j \le m_1 - 1\};\ (1 \le i \le 2),\\
W \cap N(a^{**}_{m_2,2}) &= \{a^{**}_{j,1} \mid 2 \le j \le m_1 - 1\}.
\end{aligned}$$

Since every two distinct vertices $u, v \in V(K_{m_1,m_2} \square P_2) \backslash W$ satisfies $\emptyset \neq W \cap N(u) \neq W \cap N(v) \neq \emptyset$, we obtain that W is a locating-dominating set of $K_{m_1,m_2} \square P_2$. $\square$

4. Conclusion

In this paper, we consider the Cartesian product between complete bipartite graphs K_{m_1,m_2} $(2 \le m_1 \le m_2)$ and a path of order two P_2, denoted by $K_{m_1,m_2} \square P_2$. We determine an exact value of the locating-dominating number of $K_{m_1,m_2} \square P_2$. In particular, we obtain that the locating-dominating number of $K_{m_1,m_2} \square P_2$ is:

$$\lambda(K_{m_1,m_2} \square P_2) = \begin{cases} 4, & m_1 = 2 \text{ and } m_2 \in \{2,3\},\\ m_2 + 1, & m_1 = 2 \text{ and } m_2 \ge 4,\\ m_2 + 2, & m_1 = 3 \text{ and } m_2 \in \{3,4\},\\ m_1 + m_2 - 2, & \text{otherwise.} \end{cases}$$

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