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NEW ENTROPY FOR PYTHAGOREAN FUZZY SET AND ITS APPLICATION IN MULTICRITERIA DECISION MAKING

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Abstract. *In this study, a new entropy measure for Pythagorean fuzzy sets is proposed. The validity of the entropy is rigorously verified through the satisfaction of key axiomatic properties such as minimality, maximality, symmetry, and resolution. Once validated, the newly developed entropy is applied to a multi-criteria decision-making (MCDM) problem using the TOPSIS method. The integration of this entropy with TOPSIS provides a robust and systematic approach to determining optimal alternatives in decision-making scenarios involving uncertainty and imprecision. The proposed method enhances the ability to model and analyze complex problems, thereby contributing a novel and effective tool to the field of fuzzy decision-making*

Keywords: Entropy, Pythagorean fuzzy, TOPSIS, MCDM

1. Introduction

Entropy, a measure of disorder or uncertainty, has found diverse applications across scientific disciplines. In information theory, engineering, and computer science, entropy serves as a fundamental concept for quantifying uncertainty and designing robust systems [1]. Its applications extend to biomedical signal processing, where it helps analyze complex, noisy data sets, particularly in cancer research [2]. Entropy has also been explored in the field of fuzzy logic and applied across various domains.

Fuzzy entropy is a measure of uncertainty in fuzzy sets and systems, providing a quantification of fuzziness [3]. Several researchers have proposed various formulations and properties of fuzzy entropy. Kruse [4] studied entropy measures for fuzzy events using fuzzy measures and integrals, presenting conditions for their existence under specific axioms. Ming and Li [3] discussed important fuzzy entropy formulae and their properties, introducing a new class of formulae for analyzing fuzzy systems. Guo [5] extended the concept to (k, q) order generalized entropy and q-order

Shannon entropy for fuzzy sets, proving that q-order Shannon fuzzy entropy is a σ-entropy on $F(X)$ when $q \geq 0.5$. Liu [6] provided an overview of fuzzy variable entropy, introducing three basic requirements and discussing the maximum entropy principle for selecting membership functions. The paper also covered cross-entropy and quadratic entropy of fuzzy variables, contributing to the broader understanding of fuzzy information theory.

Recent research has focused on developing new entropy formulas for intuitionistic fuzzy sets (IFS). Zhu and Li [7] proposed a novel axiomatical entropy definition that considers both information amount and reliability of IFS. Wu [8] introduced a revised axiomatic definition and a new entropy formula, applying it to multiple attribute decision making. Joshi and Kumar [9] presented an intuitionistic fuzzy entropy of order-α, which accounts for uncertainty and hesitancy degrees of IF sets, and demonstrated its application in decision-making for insurance companies. Gao [10] addressed deficiencies in existing approaches by proposing a revised axiomatic definition and structural formula for intuitionistic fuzzy entropy that better reflect-the fuzziness and intuitionism of IFS. These studies collectively contribute to the advancement of the understanding and application of entropy in intuitionistic fuzzy set theory, offering improved measures for uncertainty quantification and decision-making processes.

Recent research has introduced several new entropy measures for Pythagorean fuzzy sets (PFS), extending the concept beyond intuitionistic fuzzy sets. Gandotra et al. [11] proposed a flexible PFS entropy measure for multi-criteria decision analysis, demonstrating its reliability through comparative analysis. Li et al. [12] developed novel PFS entropy and cross-entropy measures based on Jensen–Shannon divergence, applying them to pattern recognition and decision-making problems. Yang and Hussain [13] introduced both probabilistic and non-probabilistic entropy measures for PFS, showcasing their effectiveness in linguistic variable selection and multi-criterion decision making. Most recently, Kashyap et al. [14] presented a trigonometric entropy measure for PFS, validating its performance using the COPRAS method. These new entropy formulations provide improved tools for handling uncertainty and vagueness in complex decision-making scenarios, offering improved precision and applicability compared to previous methods.

Recent research has increasingly focused on the application of Pythagorean fuzzy entropy in conjunction with TOPSIS to address multi-criteria decision-making (MCDM) problems. For instance, [15] proposed a TOPSIS-based methodology using Pythagorean fuzzy sets (PFS) and an entropy-based weighting scheme. [16] introduced a novel entropy measure tailored for PFS and applied it to rank automotive companies. [17] developed a new entropy function for PFS and integrated it with TOPSIS for group decision-making using linguistic assessments. Similarly, [18] presented a robust fuzzy entropy measure for PFS and demonstrated its effectiveness in supplier selection through an extended TOPSIS framework.

In this study, we propose a new entropy function for Pythagorean fuzzy sets, which is then combined with the TOPSIS method to offer a powerful and flexible approach for solving multi-criteria decision-making problems. This newly introduced entropy is constructed using a trigonometric-based formulation that satisfies key axiomatic properties, thereby contributing a novel mathematical framework to the field.

2. Some Concepts

Definition 2.1. *[19] Let X be a universe of discourse. A Pythagorean fuzzy set (PFS) S in X is defined as:*

$$S = \{\langle x, \mu_S(x), \nu_S(x)\rangle \mid x \in X\},$$

where the functions $\mu_S : X \to [0,1]$ and $\nu_S : X \to [0,1]$ denote the membership and non-membership degrees of the element $x \in X$ in the set S, respectively, and they satisfy the condition:

$$\mu_S(x)^2 + \nu_S(x)^2 \leq 1 \quad \text{for all } x \in X.$$

The value

$$\delta_S(x) = \sqrt{1 - \mu_S(x)^2 - \nu_S(x)^2}$$

is called the indeterminacy degree of the element x in the set S.

Definition 2.2. *[20,21] Let X be a universe of discourse. Consider three Pythagorean fuzzy sets defined on X, namely $S = \{\langle x, \mu_S(x), \nu_S(x)\rangle \mid x \in X\}$, $S_1 = \{\langle x, \mu_{S_1}(x), \nu_{S_1}(x)\rangle \mid x \in X\}$, and $S_2 = \{\langle x, \mu_{S_2}(x), \nu_{S_2}(x)\rangle \mid x \in X\}$. The corresponding operations are defined as follows:*

(1) Union:

$$S_1 \cup S_2 = \{\langle x, \max(\mu_{S_1}(x), \mu_{S_2}(x)), \min(\nu_{S_1}(x), \nu_{S_2}(x))\rangle \mid x \in X\}.$$

(2) Intersection:

$$S_1 \cap S_2 = \{\langle x, \min(\mu_{S_1}(x), \mu_{S_2}(x)), \max(\nu_{S_1}(x), \nu_{S_2}(x))\rangle \mid x \in X\}.$$

(3) Complement:

$$S^C = \{\langle x, \nu_S(x), \mu_S(x)\rangle \mid x \in X\}.$$

(4) Algebraic sum:

$$S_1 \oplus S_2 = \left\{\langle x, \sqrt{\mu^2_{S_1}(x) + \mu^2_{S_2}(x) - \mu^2_{S_1}(x)\mu^2_{S_2}(x)}, \nu_{S_1}(x)\nu_{S_2}(x)\rangle \mid x \in X\right\}.$$

The definition of entrophy for the pythagorean fuzzification function is based on article [13].

Definition 2.3. *A function $E : \mathrm{PFS}(X) \to [0,1]$ is said to be an* entropy *on the set of Pythagorean fuzzy sets $\mathrm{PFS}(X)$ if it satisfies the following properties:*

(1) ***Minimality:*** *$E(S) = 0$ if and only if S is a crisp set.*
(2) ***Maximality:*** *$E(S) = 1$ if for all $x \in X$, the membership and non-membership degrees of S satisfy*

$$\mu_S = \vartheta_S = \delta_S = \frac{1}{\sqrt{3}}.$$

(3) ***Resolution:*** *$E(S) \leq E(K)$ if S is crisper than K, that is, $\forall x \in X$, $\mu_S(x) \leq \mu_K(x)$ and $\nu_S(x) \leq \nu_K(x)$ for $\max(\mu_K(x), \nu_K(x)) \leq \frac{1}{\sqrt{3}}$ and $\mu_S(x) \geq \mu_K(x), \nu_S(x) \geq \nu_K(x)$ for $\min(\mu_K(x), \nu_K(x)) \geq \frac{1}{\sqrt{3}}$.*

(4) ***Symmetry:*** *$E(S) = E(S^c)$, where S^c denotes the complement of S.*

Theorem 2.4. *[13] If S is crisper than K in the axiom Resolution, then we have the following inequality:*

$$\left(\mu_S(x) - \frac{1}{\sqrt{3}}\right)^2 + \left(\nu_S(x) - \frac{1}{\sqrt{3}}\right)^2 + \left(\delta_S(x) - \frac{1}{\sqrt{3}}\right)^2 \geq$$
$$\left(\mu_K(x) - \frac{1}{\sqrt{3}}\right)^2 + \left(\nu_K(x) - \frac{1}{\sqrt{3}}\right)^2 + \left(\delta_K(x) - \frac{1}{\sqrt{3}}\right)^2.$$

3. Result and Discussion

3.1. *Proposed New Entropy Pythagorean Fuzzy*

Motivated by previous studies on entropy measures, several researchers have proposed entropy functions based on trigonometric formulations. This choice may stem from the inherent property of trigonometric functions whose values are naturally bounded within the interval $[-1, 1]$, thereby ensuring stability and boundedness of the entropy measure. Following this idea, we define the entropy of a Pythagorean fuzzy set S as follows.

Definition 3.1. *Let X be the universe of discourse, and let S be a Pythagorean fuzzy set defined on X, with membership $\mu(x)$, non-membership $\nu(x)$, and indeterminacy $\delta(x)$. Define a function $E(S) : \mathrm{PFS}(X) \to [0, 1]$ for all $x_i \in X$ as follows.*

$$E(S) = \frac{1}{n}\sum_{i=1}^{n} \sin\left[\left(1 - \frac{(\mu^2(x_i) - \nu^2(x_i))^2 + (\nu^2(x_i) - \delta^2(x_i))^2 + (\delta^2(x_i) - \mu^2(x_i))^2}{2}\right)\frac{\pi}{2}\right] \tag{3.1}$$

Theorem 3.2. *The function given in Definition 3.1 is a valid Pythagorean fuzzy entropy.*

Proof. Let X be the universe of discourse, and S be a Pythagorean fuzzy set defined on X, with membership $\mu(x)$, non-membership $\nu(x)$, and indeterminacy $\delta(x) = \sqrt{1 - \mu^2(x) - \nu^2(x)}$. The entropy function is defined as follows:

$$E(S) = \frac{1}{n}\sum_{i=1}^{n} \sin\left[\left(1 - \frac{\left(\mu^2(x_i) - \nu^2(x_i)\right)^2 + \left(\nu^2(x_i) - \delta^2(x_i)\right)^2 + \left(\delta^2(x_i) - \mu^2(x_i)\right)^2}{2}\right)\frac{\pi}{2}\right].$$

Let us denote:

$$A_i = \frac{(\mu^2(x_i) - \nu^2(x_i))^2 + (\nu^2(x_i) - \delta^2(x_i))^2 + (\delta^2(x_i) - \mu^2(x_i))^2}{2},$$

then

$$E(S) = \frac{1}{n}\sum_{i=1}^{n} \sin\left[(1 - A_i)\frac{\pi}{2}\right].$$

Prove of all axioms are given below.

1. Minimality

If S is a crisp set, then for each x_i, $(\mu(x_i), \nu(x_i)) \in \{(1,0),(0,1)\}$ and $\delta(x_i) = 0$. Then the squared values are either 1 or 0. For case $\mu^2 = 1$, $\nu^2 = 0$, $\delta^2 = 0$, we get:

$$A_i = \frac{(1-0)^2 + (0-0)^2 + (0-1)^2}{2} = \frac{1+0+1}{2} = 1.$$

So,

$$E_i = \sin\left((1-1)\frac{\pi}{2}\right) = \sin(0) = 0.$$

Hence, $E(S) = 0$. For case $\mu^2 = 0$, $\nu^2 = 1$, $\delta^2 = 0$ are similar.

2. Maximality

Entropy is maximal when the degrees are equally balanced:

$$\mu^2 = \nu^2 = \delta^2 = \frac{1}{3}, \quad \text{so that } \mu = \nu = \delta = \frac{1}{\sqrt{3}}.$$

Then:

$$A_i = \frac{0+0+0}{2} = 0, \quad E_i = \sin\left((1-0)\frac{\pi}{2}\right) = \sin\left(\frac{\pi}{2}\right) = 1.$$

Hence, $E(S) = 1$.

3. Resolution

Suppose S is crisper than K in the sense of Resolution, that is:

$$\mu_S(x) \leq \mu_K(x),\ \ \nu_S(x) \leq \nu_K(x) \text{ for } \max(\mu_K(x), \nu_K(x)) \leq \frac{1}{\sqrt{3}},$$
$$\mu_S(x) \geq \mu_K(x),\ \ \nu_S(x) \geq \nu_K(x) \text{ for } \min(\mu_K(x), \nu_K(x)) \geq \frac{1}{\sqrt{3}}.$$

Base on Theorem 2.4, the following inequality holds:

$$\left(\mu_S(x) - \frac{1}{\sqrt{3}}\right)^2 + \left(\nu_S(x) - \frac{1}{\sqrt{3}}\right)^2 + \left(\delta_S(x) - \frac{1}{\sqrt{3}}\right)^2 \geq$$
$$\left(\mu_K(x) - \frac{1}{\sqrt{3}}\right)^2 + \left(\nu_K(x) - \frac{1}{\sqrt{3}}\right)^2 + \left(\delta_K(x) - \frac{1}{\sqrt{3}}\right)^2.$$

This means S is closer to the crisp point and thus more certain, which implies:

$$A_i^S > A_i^K \Rightarrow (1 - A_i^S) < (1 - A_i^K) \Rightarrow \sin((1 - A_i^S)\frac{\pi}{2}) < \sin((1 - A_i^K)\frac{\pi}{2}).$$

Hence, $E(S) < E(K)$.

4. Symmetry

Because $S^C = \{x, \vartheta_S(x), \mu_S(x), \forall x \in X\}$, so:

$$\begin{aligned} E(S^C) &= \frac{1}{n}\sum_{i=1}^{n} \sin\left[\left(1 - \frac{1}{2}\Big(\nu_S^2(x_i) - \mu_{S_2}^2(x_i)\Big)^2\right.\right. \\ &+ \left.\left.\Big(\mu_{S_2}^2(x_i) - \delta_{S_2}^2(x_i)\Big)^2 + \Big(\delta_{S_2}^2(x_i) - \nu_{S_2}^2(x_i)\Big)^2\right)\frac{\pi}{2}\right], \\ &= \frac{1}{n}\sum_{i=1}^{n} \sin\left[\left(1 - \frac{1}{2}\Big(\mu_S^2(x_i) - \nu_{S_2}^2(x_i)\Big)^2\right.\right. \\ &+ \left.\left.\Big(\nu_{S_2}^2(x_i) - \delta_{S_2}^2(x_i)\Big)^2 + \Big(\delta_{S_2}^2(x_i) - \mu_{S_2}^2(x_i)\Big)^2\right)\frac{\pi}{2}\right], \\ &= E(S). \end{aligned}$$

So we get

$$E(S^C) = E(S). \qquad \square$$

Theorem 3.3. *Let X be a universe of discourse, and let $S_1, S_2 \in \mathrm{PFS}(X)$. Suppose that $S_1 \subseteq S_2$ or $S_2 \subseteq S_1$. Given the entropy function:*

$$E(S) = \frac{1}{n}\sum_{i=1}^{n} \sin\left[\left(1 - \frac{(\mu^2(x_i) - \nu^2(x_i))^2 + (\nu^2(x_i) - \delta^2(x_i))^2 + (\delta^2(x_i) - \mu^2(x_i))^2}{2}\right)\frac{\pi}{2}\right],$$

the following relationship holds:

$$E(S_1 \cup S_2) + E(S_1 \cap S_2) = E(S_1) + E(S_2).$$

Proof. Let $S_1, S_2 \in \mathrm{PFS}(X)$ be two Pythagorean fuzzy sets over the universe $X = \{x_1, x_2, \cdots, x_n\}$. Suppose that $S_1 \subseteq S_2$ or $S_2 \subseteq S_1$. Then for each $x_i \in X$, either:

$$\mu_1(x_i) \le \mu_2(x_i) \text{ and } \nu_1(x_i) \ge \nu_2(x_i), \quad \text{or vice versa.}$$

Based on this, it follows that:

$$\begin{aligned} E(S_1 \cup S_2) &= \frac{1}{n}\sum_{i=1}^{n} \sin\left[\left(1 - \frac{1}{2}\Big(\mu_{S_1\cup S_2}^2(x_i) - \nu_{S_1\cup S_2}^2(x_i)\Big)^2\right.\right. \\ &+ \left.\left.\Big(\nu_{S_1\cup S_2}^2(x_i) - \delta_{S_1\cup S_2}^2(x_i)\Big)^2 + \Big(\delta_{S_1\cup S_2}^2(x_i) - \mu_{S_1\cup S_2}^2(x_i)\Big)^2\right)\frac{\pi}{2}\right], \\ &= \frac{1}{n}\sum_{i=1}^{n} \sin\left[\left(1 - \frac{1}{2}\Big(\mu_{S_2}^2(x_i) - \nu_{S_2}^2(x_i)\Big)^2\right.\right. \\ &+ \left.\left.\Big(\nu_{S_2}^2(x_i) - \delta_{S_2}^2(x_i)\Big)^2 + \Big(\delta_{S_2}^2(x_i) - \mu_{S_2}^2(x_i)\Big)^2\right)\frac{\pi}{2}\right], \\ &= E(S_2). \end{aligned}$$

$$
\begin{aligned}
E(S_1 \cap S_2) = {} & \frac{1}{n}\sum_{i=1}^{n} \sin\Bigg[\Big(1 - \frac{1}{2}\Big(\mu^2_{S_1\cap S_2}(x_i) - \nu^2_{S_1\cap S_2}(x_i)\Big)^2 \\
& + \Big(\nu^2_{S_1\cap S_2}(x_i) - \delta^2_{S_1\cap S_2}(x_i)\Big)^2 + \Big(\delta^2_{S_1\cap S_2}(x_i) - \mu^2_{S_1\cap S_2}(x_i)\Big)^2\Big)\frac{\pi}{2}\Big)\Bigg], \\
= {} & \frac{1}{n}\sum_{i=1}^{n} \sin\Bigg[\Big(1 - \frac{1}{2}\Big(\mu^2_{S_1}(x_i) - \nu^2_{S_1}(x_i)\Big)^2 \\
& + \Big(\nu^2_{S_1}(x_i) - \delta^2_{S_1}(x_i)\Big)^2 + \Big(\delta^2_{S_1}(x_i) - \mu^2_{S_1}(x_i)\Big)^2\Big)\frac{\pi}{2}\Big)\Bigg], \\
= {} & E(S_1).
\end{aligned}
$$

So we get:

$$E(S_1 \cup S_2) + E(S_1 \cap S_2) = E(S_1) + E(S_2). \qquad \square$$

3.2. *Pythagorean Fuzzy Multicriteria Dicesion Making Based on New Entropy*

We will use this entropy as a method to solve a multi-criteria decision-making problem by integrating it with the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) approach. TOPSIS is a well-known multi-criteria decision-making (MCDM) method. The overall procedure that combines the entropy and TOPSIS methods is illustrated in Figure 1.

While detail of TOPSIS decision process follows the following steps.

(1) **Construct the Pythagorean Fuzzy Decision Matrix (PFDM)**
Let the decision matrix $M = (r_{ij})_{m\times n}$ represent a multi-criteria decision-making (MCDM) problem, where each element r_{ij} is a Pythagorean fuzzy number associated with alternative A_i and criterion C_j. The matrix is given as:

$$M = \begin{pmatrix} (\mu_{11}, \nu_{11}, \delta_{11}) & (\mu_{12}, \nu_{12}, \delta_{12}) & \cdots & (\mu_{1n}, \nu_{1n}, \delta_{1n}) \\ (\mu_{21}, \nu_{21}, \delta_{21}) & (\mu_{22}, \nu_{22}, \delta_{22}) & \cdots & (\mu_{2n}, \nu_{2n}, \delta_{2n}) \\ \vdots & \vdots & \ddots & \vdots \\ (\mu_{m1}, \nu_{m1}, \delta_{m1}) & (\mu_{m2}, \nu_{m2}, \delta_{m2}) & \cdots & (\mu_{mn}, \nu_{mn}, \delta_{mn}) \end{pmatrix}. \tag{3.2}$$

(2) **Determine the Weights of Criteria Using Entropy**
Given entropy values e_j for each criterion j, the normalized weight w_j is calculated as:

$$w_j = \frac{1 - e_j}{\sum_{j=1}^{n}(1 - e_j)}, \tag{3.3}$$

where $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$.

(3) **Calculate the Distance to Ideal Solutions**
For each alternative A_i, compute the Euclidean distances to the positive ideal solution (PIS) and negative ideal solution (NIS) as follows.

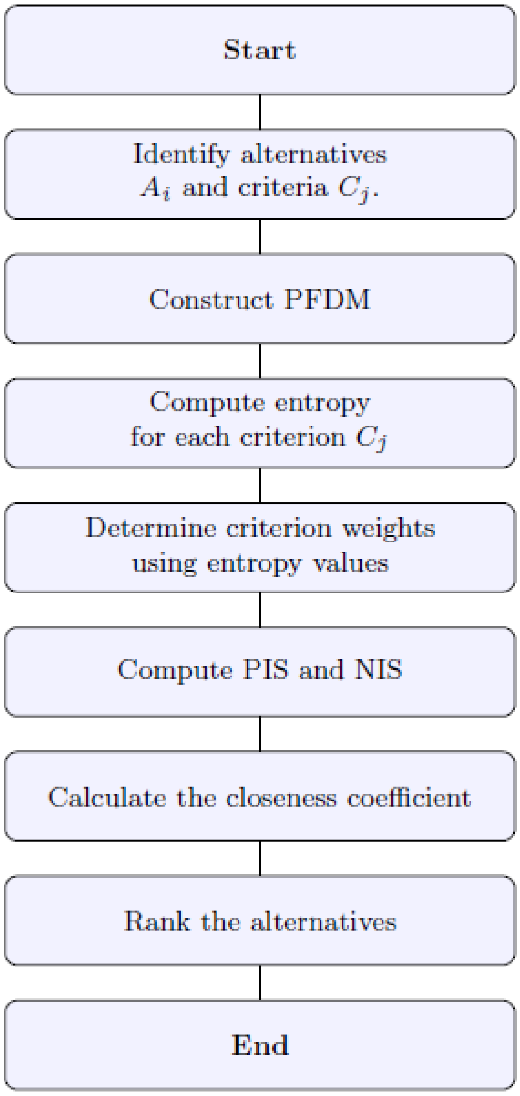

Figure. 1. TOPSIS-Entropy PFS Flowchart

Distance to PIS:

$$D_i^+ = \sqrt{\frac{1}{2}\sum_{j=1}^{n} w_j \left[(1-\mu_{ij}^2)^2 + (\nu_{ij}^2)^2 + (1-\mu_{ij}^2-\nu_{ij}^2)^2\right]}. \tag{3.4}$$

Distance to NIS:

$$D_i^- = \sqrt{\frac{1}{2}\sum_{j=1}^{n} w_j \left[(\mu_{ij}^2)^2 + (1-\nu_{ij}^2)^2 + (1-\mu_{ij}^2-\nu_{ij}^2)^2\right]}. \tag{3.5}$$

(4) **Compute the Relative Closeness Coefficient**
Calculate the closeness coefficient of each alternative using:

$$C_i = \frac{D_i^-}{D_i^+ + D_i^-}. \tag{3.6}$$

A higher C_i indicates a better alternative.

(5) **Rank the Alternatives**
Rank all alternatives in descending order based on the values of C_i. The alternative with the highest C_i is considered the most preferable.

TOPSIS combined with fuzzy entropy can be utilized to solve decision-making problems involving many criteria (multi-criteria decision making). For example, it can be used to determine the most suitable location for a final waste disposal site (landfill) in a certain region. The selection of a landfill site generally considers the following criteria in Table 1.

Table 1. Selection Criteria for Landfill Site

Symbol	**Criteria**	**Description**
$C1$	Topography	The site should be relatively flat or gently sloped to facilitate the operation of heavy machinery and proper drainage.
$C2$	Distance from Water Sources	The site must be far from rivers, lakes, or groundwater wells to prevent water contamination.
$C3$	Distance from Residential Areas	The site should be sufficiently far from residential areas to reduce odor, disease vectors, and social resistance.

Suppose that there are four alternative locations to choose from, denoted as $\{A_1, A_2, A_3, A_4\}$. Based on these alternatives, the Pythagorean fuzzy membership values for each alternative are determined according to the given criteria. These values are provided in the following table, this table actually represents Pythagorean fuzzy decision matrices as given in Equation (3.2).

Table 2. Pythagorean Fuzzy Membership for Each Alternative and Criterion

Alternatives	C1 (μ, ν, δ)	C2 (μ, ν, δ)	C3 (μ, ν, δ)
A1	(0.51, 0.43, 0.555)	(0.55, 0.45, 0.495)	(0.45, 0.35, 0.675)
A2	(0.62, 0.33, 0.5067)	(0.65, 0.41, 0.4094)	(0.61, 0.55, 0.3254)
A3	(0.48, 0.25, 0.7071)	(0.45, 0.38, 0.6531)	(0.35, 0.45, 0.675)
A4	(0.72, 0.51, 0.2215)	(0.68, 0.45, 0.3351)	(0.75, 0.30, 0.3475)

The entropy of each criterion is calculated using Equation 3.1, and subsequently, the weight for each criterion is determined using Equation 3.3. The results of these calculations are presented in the following Table 3.

Table 3. Entropy and Weight for Each Criterion

Criteria	**Entropy** E_j	**Weight** w_j
C1	0.9840	0.3886
C2	0.9943	0.1383
C3	0.9805	0.4731

Using the results obtained in Table 3, the distances to the ideal solutions are then calculated: the distance to the positive ideal solution using Equation 3.4, and the distance to the negative ideal solution using Equation 3.5. The results of these calculations are presented in Table 4 as follows.

Table 4. Distance to Positive and Negative Ideal Solutions

Alternative	**Distance to PIS** (D_i^+)	**Distance to NIS** (D_i^-)
A1	0.6986	0.7543
A2	0.5499	0.6911
A3	0.7665	0.7893
A4	0.4172	0.7307

Based on the results in Table 4, the relative closeness coefficient is then determined using Equation 3.6. The alternatives are ranked according to the highest value of the relative closeness coefficient. The results of the calculation and the corresponding rankings are shown in the following Table 5.

Table 5. Final Results of TOPSIS with Closeness Coefficient and Ranking

Alternative	D_i^+	D_i^-	C_i (Closeness Coefficient)	**Rank**
A4	0.4172	0.7307	0.6365	1
A2	0.5499	0.6911	0.5567	2
A1	0.6986	0.7543	0.5190	3
A3	0.7665	0.7893	0.5076	4

Based on the preference values, alternative A_4 is the most preferred option, followed by A_2, then A_1 and the last A_3.

4. Conclusion

This paper introduces a novel entropy function for Pythagorean fuzzy sets and confirms its validity through a comprehensive axiomatic analysis. The validated entropy measure is then successfully integrated with the TOPSIS method to address multi-criteria decision-making problems. The results demonstrate that the proposed approach effectively captures uncertainty within decision-making environments and provides reliable rankings of alternatives. This study contributes to the advancement of fuzzy set theory and decision science by offering a mathematically sound and practically applicable entropy-based MCDM framework.

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