MATHEMATICAL ANALYSIS OF SEXUAL VIOLENCE DYNAMICS WITH RECIDIVIST PERPETRATORS

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Abstract. Sexual violence remains a serious social issue with far-reaching consequences for both victims and society at large. To capture the dynamics of its spread, this study develops a compartmental mathematical model that divides the population into four subgroups: susceptible individuals (S), perpetrators (V), punished offenders (P), and rehabilitated individuals (R). This model incorporates a recidivist parameter, which signifies the propensity of individuals who have been convicted of sexual violence to reoffend, that is transition from P to V. The analysis includes the determination of equilibrium points, computation of the basic reproduction number using the Next Generation Matrix approach, and assessment of local stability through eigenvalue evaluation of the Jacobian matrix. the violence-free equilibrium is locally asymptotically stable if the basic reproduction number $R_0 < 1$, whereas the endemic equilibrium is locally asymptotically stable when $R_0 > 1$. In addition, the presence of recidivist perpetrators increases the basic reproduction number, thereby amplifying the likelihood of sustained sexual violence within the population. In particular, the relapse rate is shown to play a critical role in destabilizing the violence-free equilibrium, underscoring the importance of addressing recidivism in prevention and intervention strategies. These findings provide new insights into the mathematical modeling of sexual violence and highlight the necessity of targeted policies to mitigate its persistence.

Keywords: Next generation matrix, Sexual violence, Stability Analysis

1. Introduction

According to the interpretation of the World Health Organization (WHO) on the subject, sexual violence can be defined as any kind of sexual act or behavior directed against an individual through coercion, irrespective of the relationship between the perpetrator and the victim or the context in which it transpires [1]. It is estimated that 30% women worldwide will be subjected to sexual violence during their lifetime. This encompasses a range of behaviors, including coerced sexual acts or attempts, non-consensual acts, and those committed through force [2]. Various studies

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on sexual and gender-based violence demonstrate diverse approaches, ranging from technological innovations to social interventions. Educational interventions through website media significantly improved adolescents' knowledge and attitudes in preventing risky sexual behavior, though the impact on actual practices was minimal [3]. Another technological innovation, an AI framework based on gait recognition from IoT devices, achieved high accuracy in detecting violence in public spaces, yet may face challenges related to privacy and implementation costs [4]. Social studies further examined the causal factors of intimate partner violence (IPV) in India and Bolivia using statistical analysis [5], and a mathematical modeling approach was developed to link domestic violence questionnaires with simulated violence scenarios, though this remains restricted to analytical aspects [6]. In [7], the SPVR model is constructed to investigate the dynamics of sexual assault in Indonesia using reported cases of violence collected by Komnas Perempuan Indonesia. According to [8], the recidivism rate for perpetrators of sexual violence is 2%. In order to address this research gap and extend the findings of previous studies [7], this study incorporates a new parameter representing recidivism into the existing model to account for repeated offenders, and analyzes how this factor affects the stability of the equilibrium points and the overall system behavior. This will facilitate a more comprehensive understanding of the dynamics of sexual violence.

2. Methods

The analytical framework of the model is constructed under the assumption that no structural changes occur within each subpopulation that would alter the basic composition of the system. This assumption is made so that the population can be considered stable enough to permit the derivation of equilibrium points. Assuming this premise, two distinct types of equilibria can be delineated: the sexual violence-free equilibrium and the endemic equilibrium. Subsequent to the formal establishment of these equilibrium points, the subsequent step involves the determination of the basic reproduction number, a pivotal epidemiological threshold parameter that reflects the potential for the continuation or eradication of the phenomenon. This determination is made by applying the Next Generation Matrix method.

Subsequent to this, a thorough examination of the stability properties of the equilibria is conducted. In order to ascertain the violence-free equilibrium, it is necessary to construct the Jacobian matrix of the system and calculate its associated eigenvalues. Concurrently, the stability of the endemic equilibrium is evaluated by implementing the general stability theorem in conjunction with the Routh–Hurwitz criterion. This approach provides a systematic method for ascertaining whether all characteristic roots of the system are have negative real parts, thereby ensuring local asymptotic stability.

Finally, in order to complement and validate the theoretical derivations, numerical simulations are carried out using the MAPLE software. These simulations not only corroborate the analytical results obtained in the earlier stages but also furnish supplementary insights into the long-term behaviors that may emerge in real-world scenarios, thereby fortifying the overall interpretation of the model.

3. Result and Discussion

3.1. *Sexual Violence Model*

The population N is divided into four subpopulations: susceptible (S), perpetrators (V), punished (P), and recovered (R). Susceptible individuals may become perpetrators of sexual violence through contact with existing perpetrators, characterized by an effective contact rate α. Some individuals, however, may directly enter the recovered class at a rate θ when they successfully resist engaging in such acts. Perpetrators who are adjudged guilty by the judicial system are transferred to the punished subpopulation at a rate δ. A notable aspect of this phenomenon is that, upon completion of their sentences, a proportion of these formerly punished individuals may re-enter the perpetrator subpopulation at a rate β, thus exhibiting recidivist tendencies. Individuals demonstrating rehabilitation and abstinence move to the recovered class at a rate ρ, while punished individuals may also recover at a rate γ. Furthermore, individuals in the recovered group may revert to the susceptible class at a rate σ due to environmental or social influences. The model also assumes a constant natural birth rate b, which adds new individuals to the susceptible class, and a natural death rate μ that applies uniformly across all subpopulations. The corresponding compartmental diagram in Figure 1 captures these dynamic transitions.
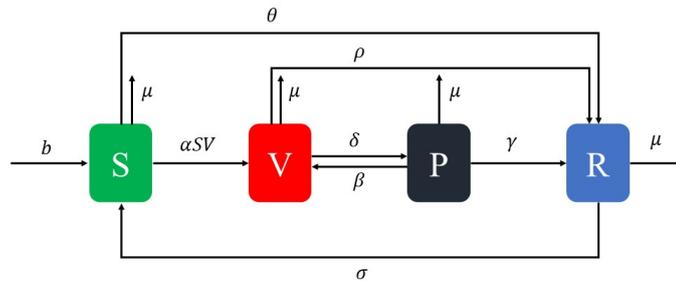

Figure. 1. Sexual violence SVPR model compartmental diagram.

Mathematically, the dynamics are represented by the following system of nonlinear differential equations in proportions. These equations are obtained by normalizing each compartment with respect to the total population, denoted by N. That is to say, the compartmental proportions are expressed as $s = S/N$, $v = V/N$, $p = P/N$, and $r = R/N$. It should be noted that the sum of the proportions, $s + v + p + r$, must always equal one. The following equation is intended to provide a comprehensive overview of the subject matter.

$$\frac{ds}{dt} = b - \alpha sv + \sigma r - (\theta + \mu)s,$$

$$\frac{dv}{dt} = \alpha sv + \beta p - (\delta + \rho + \mu)v,$$

$$\frac{dp}{dt} = \delta v - (\beta + \gamma + \mu)p,$$

$$\frac{dr}{dt} = \rho v + \gamma p + \theta s - (\sigma + \mu)r,$$

(3.1)

with initial value of s is in $(0, 1]$ and other variable is in $[0, 1]$. The model parameters are outlined in Table 1.

Table 1. Sexual Violence Model Parameter

Parameter	Description	Value
b	Natural birth rate	0.00329 *
α	Effective contact rate that can spread sexual violence	0.00082 *
δ	Punished rate of perpetrators	0.04424 **
β	Recidivist rate	0.00200 *
ρ	Perpetrators recovery rate	0.00442 **
γ	Rate at which punished individuals become recovered	0.05308 **
θ	Rate at which susceptible individuals become recovered	0.00004 **
σ	Rate at which recovered individuals become susceptible	0.46566 **
μ	Natural death rate	0.00329 *

* Estimated from empirical survey data.
** Assumed from previous studies.

We let $s(t), v(t), p(t), r(t)$ denote the proportions of the total population in each compartment. All parameters are therefore expressed as per unit time.

3.2. *Sexual Violence Equilibria*

In order to ascertain the two equilibrium states, namely the sexual violence-free and endemic equilibria, it is necessary to set each equation in System (3.1) to zero, under the assumption that $s = s^0$; $v = v^0$; $p = p^0$; $r = r^0$ [9]. The sexual violence-free equilibrium is defined as a state in which there is an absence of perpetrators within the population. This can be expressed as $v^0 = 0$ [10,11]. Consequently, no individuals are assigned to the punished compartment, implying $p^0 = 0$. Therefore, the equilibrium point for the gender-based sexual violence-free state is given by The following text is intended to provide a comprehensive overview of the subject matter.

$$E_0 = (s^0, v^0, p^0, r^0) = \left(\frac{b(\sigma + \mu)}{\mu(\theta + \mu + \sigma)}, 0, 0, \frac{b\theta}{\mu(\theta + \mu + \sigma)} \right).$$

(3.2)

The basic reproduction number, denoted by R_0, is derived at the sexual violence-free equilibrium. It quantifies the potential for sexual violence transmission within a population that is fully susceptible to such acts, representing the anticipated number of secondary perpetrators who could be generated by a primary perpetrator [12]. The value of R_0 is obtained using the Next-Generation Matrix method, as outlined in [13,14]. This method incorporates the dynamics of the v and p subpopulations.

$$\frac{dv}{dt} = \alpha sv + \beta p - (\delta + \rho + \mu)v,$$
$$\frac{dp}{dt} = \delta v - (\beta + \gamma + \mu)p. \tag{3.3}$$

The value of R_0 is derived from the largest eigenvalue of the next generation matrix $K = FV^{-1}$, where F is the matrix of partial derivatives that represent the rate of new infections in each group, and V is the matrix of partial derivatives describing the rate of transition or movement of individuals between groups. Thus,

$$F = \begin{bmatrix} \frac{\alpha b(\theta+\mu)}{\mu(\sigma+\mu+\theta)} & 0 \\ 0 & 0 \end{bmatrix}, \qquad V = \begin{bmatrix} \delta + \rho + \mu & -\beta \\ -\delta & \beta + \gamma + \mu \end{bmatrix}. \tag{3.4}$$

We can calculate

$$K = FV^{-1} = \begin{bmatrix} \frac{\alpha b(\sigma+\mu)(\beta+\gamma+\mu)}{\mu(\mu+\sigma+\theta)\Lambda} & \frac{\alpha b\beta(\sigma+\mu)}{\mu(\mu+\sigma+\theta)\Lambda} \\ 0 & 0 \end{bmatrix}, \tag{3.5}$$

where $\Lambda = (\rho + \delta + \mu)(\beta + \gamma + \mu) - \beta\delta$. Therefore, we have the R_0 of System (3.1) is:

$$R_0 = \frac{\alpha b(\sigma + \mu)(\beta + \gamma + \mu)}{\mu(\mu + \sigma + \theta)((\rho + \delta + \mu)(\beta + \gamma + \mu) - \beta\delta)}. \tag{3.6}$$

The endemic equilibrium point for gender-based sexual violence is reached when individuals who commit sexual violence continue to exist within a community. This means that $v > 0$. Let $s = s^*, v = v^*, p = p^*$, and $r = r^*$. Then, we can derive the equilibrium point for sexual violence as follows:

$$E_* = (s^*, v^*, p^*, r^*), \tag{3.7}$$

where:

$$s^* = \frac{(\rho + \delta + \mu)(\beta + \gamma + \mu) - \beta\delta}{\alpha(\beta + \gamma + \mu)},$$
$$v^* = \frac{\alpha b(\sigma + \mu)(\beta + \gamma + \mu) - \mu(\mu + \sigma + \theta)\big((\delta + \rho + \mu)(\beta + \gamma + \mu) - \beta\delta\big)}{\alpha\mu\big((\delta + \rho + \mu)(\beta + \gamma + \mu) - \beta\delta + \sigma(\beta + \delta + \gamma + \mu)\big)},$$
$$p^* = \frac{\delta}{\beta + \gamma + \mu}v^*,$$
$$r^* = \frac{(\alpha\delta\gamma + \alpha\rho(\beta + \gamma + \mu))v^* + \theta((\rho + \delta + \mu)(\beta + \gamma + \mu) - \beta\delta)}{\alpha(\sigma + \mu)(\beta + \gamma + \mu)}.$$

The existence of the endemic equilibrium requires that v^* be positive. Mathematically, this condition ensures that the inflow of offenders—driven by the effective contact rate and the rate of recidivist—exceeds the combined outflow due to punished

and recovered perpetrators, and also natural death. In other words, the persistence of violent individuals in the population depends on whether the generation rate of new offenders surpasses the removal processes.

A formal relationship between the positivity of v^* and the threshold parameter R_0 will be established in Subsection 3.3, where it will be shown that $v^* > 0$ corresponds to the condition $R_0 > 1$.

3.3. *Stability Analysis*

We assess the stability of the equilibrium points in model (3.1) through linearization with the Jacobian matrix [15].

$$J = \begin{bmatrix} -\alpha v - (\theta + \mu) & -\alpha s & 0 & \sigma \\ \alpha v & \alpha s - (\delta + \rho + \mu) & \beta & 0 \\ 0 & \delta & -(\beta + \gamma + \mu) & 0 \\ \theta & \rho & \gamma & -(\sigma + \mu) \end{bmatrix}. \tag{3.8}$$

Theorem 3.1. *The sexual violence-free equilibrium point (E_0) is locally asymptotically stable if:*

$$R_0 < 1, \tag{3.9}$$

$$\mu(\theta + \mu + \sigma)[(\delta + \rho + \mu) + (\beta + \gamma + \mu)] - \alpha b(\sigma + \mu) < 0.$$

Proof. Substituting Eq. (3.2) into Eq. (3.8) yields the Jacobian at the sexual violence-free equilibrium point.

$$J_{E_0} = \begin{bmatrix} -a_1 & -a_2 & 0 & \sigma \\ 0 & a_2 - a_3 & \beta & 0 \\ 0 & \delta & -a_4 & 0 \\ \theta & \rho & \gamma & -a_5 \end{bmatrix}. \tag{3.10}$$

where $a_1 = \theta + \mu, a_2 = \dfrac{\alpha b(\sigma + \mu)}{\mu(\theta + \mu + \sigma)}, a_3 = \delta + \rho + \mu, a_4 = \beta + \gamma + \mu, a_5 = \sigma + \mu.$
We derive the characteristic equation for J_{E_0} that is

$$[\lambda^2 + b_1\lambda + b_2][\lambda^2 + b_3\lambda + b_4] = 0, \tag{3.11}$$

where $b_1 = a_1 + a_5, b_2 = a_1 a_5 - \sigma\theta, b_3 = (a_3 - a_2) + a_4, b_4 = a_4(a_3 - a_2) - \beta\delta.$ Equilibrium point (E_0) is said to be asymptotically stable if all the eigenvalues of jacobian matrix evaluated at that point have negative real parts. Using Routh-Hurwitz criterion, all roots of Eq. (3.11) are negative if and only if all the following is true [16]:

(i) $b_3 = a_1 + a_5 > 0 \iff \theta + \sigma + 2\mu > 0.$
(ii) $b_3 = (a_1 a_5 - \sigma\theta) > 0 \iff \mu(\sigma + \theta + \mu) > 0.$
(iii) $b_1 = ((a_3 - a_2) + a_4) > 0.$

$$\iff (\delta + \rho + \mu) - \left(\frac{\alpha b(\sigma + \mu)}{\mu(\theta + \mu + \sigma)}\right) + (\beta + \gamma + \mu) > 0,$$

$$\iff \mu(\theta + \mu + \sigma)[(\delta + \rho + \mu) + (\beta + \gamma + \mu)] - \alpha b(\sigma + \mu) > 0;$$

$$\text{if } \mu(\sigma + \theta + \mu) > 0.$$

(iv) $b_2 = (a_4(a_3 - a_2) - \beta\delta) > 0$.

$$\iff (\beta + \gamma + \mu)\left[(\delta + \rho + \mu) - \left(\frac{\alpha b(\sigma + \mu)}{\mu(\theta + \mu + \sigma)}\right)\right] - \beta\delta > 0,$$

$$\iff \frac{\alpha b(\sigma + \mu)(\beta + \gamma + \mu)}{\mu(\mu + \sigma + \theta)((\rho + \delta + \mu)(\beta + \gamma + \mu) - \beta\delta)} < 1 \iff R_0 < 1.$$

The veracity of (i) and (ii) is evident, as all utilized parameters are positive. Therefore, it can be concluded that the equilibrium point (E_0) is asymptotically stable if conditions (iii) and (iv) are met. □

Theorem 3.2. *The endemic equilibrium point (E_*) is locally asymptotically stable if $R_0 > 1$ and all eigenvalues of the Jacobian matrix at this point are negative.*

Proof. For v^* in Eq. (3.7), it can be rewritten as follows:

$$v^* = \frac{(\mu + \sigma + \theta)\big((\delta + \rho + \mu)(\beta + \gamma + \mu) - \beta\delta\big)(R_0 - 1)}{\alpha\big((\delta + \rho + \mu)(\beta + \gamma + \mu) - \beta\delta + \sigma(\beta + \delta + \gamma + \mu)\big)}. \tag{3.12}$$

From this equation, we can conclude that, for $v^* > 0$, R_0 must be greater than 1. Now, the substitution of Eq. (3.7) for Jacobian in Eq. (3.8) was performed, resulting in Eq. (3.13):

$$J_{E_*} = \begin{bmatrix} -g_1 & -\alpha s^* & 0 & \sigma \\ \alpha v^* & -g_2 & \beta & 0 \\ 0 & \delta & -g_3 & 0 \\ \theta & \rho & \gamma & -g_4 \end{bmatrix}, \tag{3.13}$$

where $g_1 = \alpha v^* + \theta + \mu, g_2 = -\alpha s^* + \delta + \rho + \mu, g_3 = (\beta + \gamma + \mu), g_4 = (\sigma + \mu)$.

It is possible to derive the polynomial characteristic of this matrix, that is,

$$P(\lambda) = \lambda^4 + h_1\lambda^3 + h_2\lambda^2 + h_3\lambda + h_4, \tag{3.14}$$

where

$h_1 = g_1 + g_2 + g_3 + g_4,$

$h_2 = g_1g_2 + g_1g_3 + g_1g_4 + g_2g_3 + g_2g_4 + g_3g_4\alpha^2 s^* v^* - \beta\delta - \sigma\theta,$

$h_3 = g_3g_4(g_1 + g_2) + (\alpha^2 s^* v^* + g_1g_2)(g_3 + g_4) - \alpha\rho\sigma v^* - \beta\delta(g_1 + g_4) - \sigma\theta(g_2 + g_3),$

$h_4 = g_1g_2g_3g_4 + \alpha^2 s^* v^* g_3g_4 - \alpha\delta\gamma\sigma v - \alpha g_3\rho\sigma v - \beta\delta g_1g_4 + \beta\delta\sigma\theta - g_2g_3\sigma\theta.$

According to the Routh-Hurwitz criterion, the real part of the eigenvalues of the matrix J_{E_*} is negative under the subsequent conditions:

$$\begin{aligned} h_1 &> 0, \\ h_1h_2 - h_3 &> 0, \\ h_1h_2h_3 - h_1^2h_4 - h_3^2 &> 0, \\ h_1h_2h_3h_4 - h_3^2h_4 - h_1^2h_4^2 &> 0. \end{aligned} \tag{3.15}$$

Thus, the equilibrium point (E_*) is asymptotically stable if satisfies conditions in (3.15). □

4. Numerical Simulation

To validate these findings, a numerical simulation was executed using the MAPLE. The parameter values and estimated initial value for variables of the model defined in equation (3.1) are presented in Table 1 and 2, respectively.

Table 2. Initial value of variables

Variable	Value
$s(0)$	0.998001
$v(0)$	0.001654
$p(0)$	0.000314
$r(0)$	0.000031

Accordingly, the determination of the basic reproductive number value is achieved by employing the parameters in Table 1 to Eq. (3.6). The resulting value is 0.016456. With the same parameter values, the sexual violence-free equilibrium point, and the condition for its stability in Eq. (3.9) is found to be:

$$E_0 = (0.9999147, 0, 0, 0.0000853), \tag{4.1}$$

$$\mu(\theta + \mu + \sigma)[(\delta + \rho + \mu) + (\beta + \gamma + \mu)] - \alpha b(\sigma + \mu) = 0.0003617 > 0. \tag{4.2}$$

Since the conditions are met, as demonstrated in Theorem 3.1, thereby ensuring the stability of the equilibrium point. This phenomenon is further elucidated in the solution graph depicted in Figure 2.

Figure 2 illustrates the numerical simulation of the model under the violence-free equilibrium state. In panel (a), the susceptible population increases steadily over time and approaches the equilibrium value. This indicates that in the absence of persistent sexual violence, almost all individuals eventually remain in the susceptible class. Panel (b) shows that the number of perpetrators decreases rapidly to zero, confirming that the perpetrator class cannot be sustained under the violence-free state. Similarly, panel (c) depicts the punished class, which initially rises due to interventions but subsequently declines to zero as perpetrators vanish from the system. Panel (d) shows the recovered class, which initially increases but ultimately diminishes to equilibrium value as there are no longer perpetrators to generate new cases. Next, by changing the α and β to 0.047792 and 0.070, respectively, with the other parameters remaining unchanged, we obtain a value of R_0 and E_*, which is:

$$R_0 = 1.491894, \tag{4.3}$$

$$E_* = (0.574242, 0.304265, 0.106518, 0.014973), \tag{4.4}$$

and for Eq. (3.15), we get:

$$\begin{aligned}
h_1 &= 0.6376972938 > 0, \\
h_1 h_2 - h_3 &= 0.05095634481 > 0, \\
h_1 h_2 h_3 - h_1^2 h_4 - h_3^2 &= 0.00007325727333 > 0, \\
h_1 h_2 h_3 h_4 - h_3^2 h_4 - h_1^2 h_4^2 &= 2.905929106 \times 10^{-10} > 0.
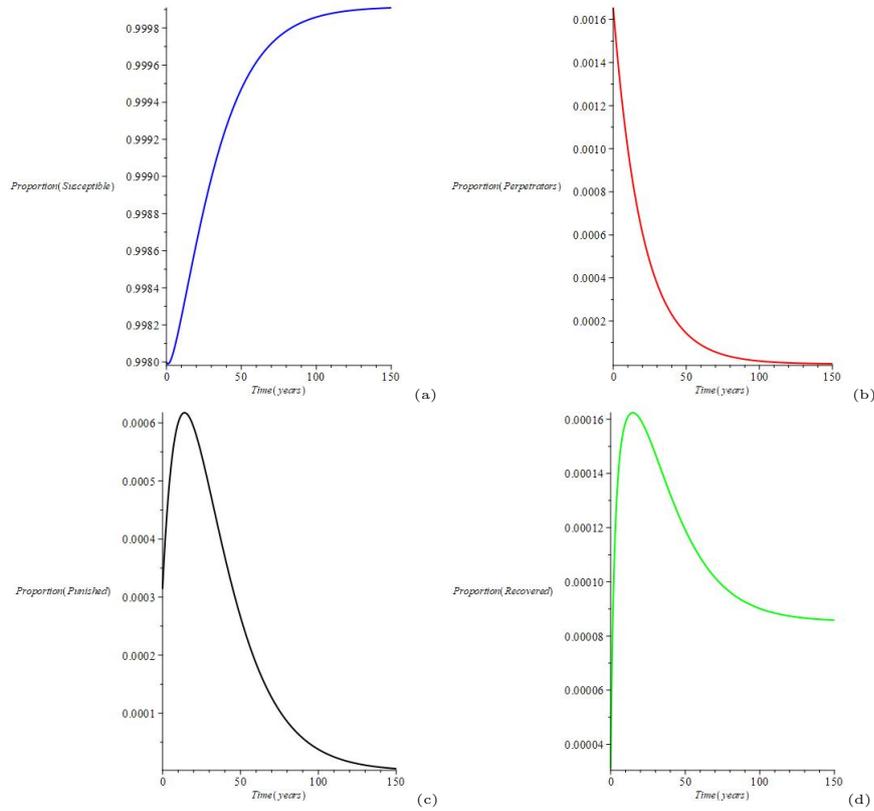\end{aligned} \tag{4.5}$$

Figure. 2. Free-sexual violence state solution graphs: (a) *Susceptible* (b) *Perpetrators* (c) *Punished* (d) *Recovered*

It has been demonstrated that the conditions for asymptotic stability are met, as evidenced by the findings presented in Theorem 3.2. To further verify this result, we computed the eigenvalues of the Jacobian matrix evaluated at the endemic equilibrium E_* using the numerical parameter values given in Table 1. The eigenvalues were found to have strictly negative real parts:

$$\lambda_1 = -0.0035, \quad \lambda_2 = -0.045, \quad \lambda_3 = -0.002, \quad \lambda_4 = -0.054,$$

confirming that the endemic equilibrium is asymptotically stable for the chosen parameter set. This phenomenon is further illustrated in the solution graph shown in Figure 3.

The endemic state solution graphs in Figure 3 illustrates the long-term dynamics of the sexual violence model. The proportion of susceptible individuals (s) decreases steadily over time, reflecting the persistent risk of transitioning into other compartments. In contrast, the proportion of perpetrators (v) exhibits a rapid increase before stabilizing at the endemic equilibrium, indicating sustained transmission and reinforcement of violent behaviors in the community. The punished compartment (p) follows a similar increasing trajectory, reflecting the implementation of intervention mechanisms and the ongoing need for punitive measures. Finally, the re-
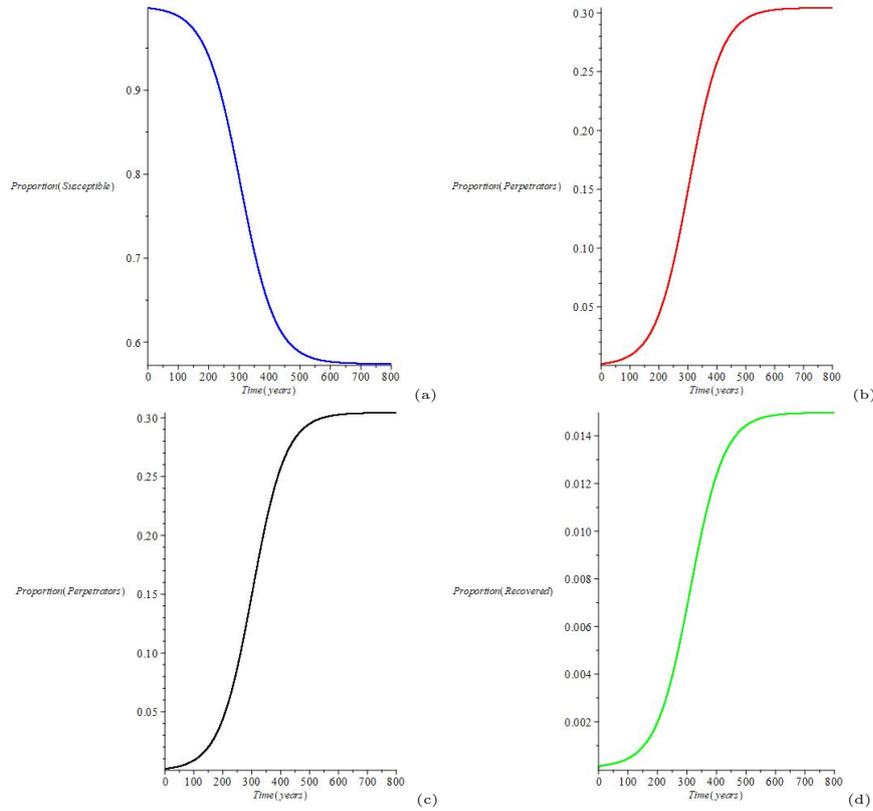
Figure. 3. Endemic sexual violence state solution graphs: (a) *Susceptible* (b) *Perpetrators* (c) *Punished* (d) *Recovered*

covered population (r) rises and stabilizes at a positive level. Overall, these trends indicate that under endemic conditions, sexual violence persists in society, with a balance maintained between susceptibility, perpetration, punishment, and recovery processes.

While Theorem 3.2 guarantees the asymptotic stability of the endemic equilibrium E^*, the simulation presented in Figure 3 represents only a single trajectory. To further support the theoretical result, additional simulations with varying initial conditions were conducted, confirming that all trajectories converge to the endemic equilibrium in Figure 4, thereby numerically verifying its local stability.

To analyze the influence of the recidivism rate, β, on the temporal evolution of the perpetrator population, numerical simulations were performed using four distinct values of β. The proportion of perpetrators, denoted by $v(t)$, was observed over a time horizon of 300 years under identical initial conditions for each scenario. This comparison allows for a clear illustration of how different levels of recidivism affect the rate at which the perpetrator population decreases over time. The corresponding trajectories are presented in Figure 5.

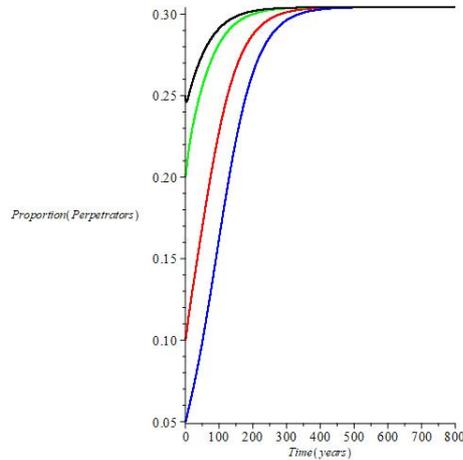In all scenarios in Figure 5, the trajectories exhibit a monotonic decline toward

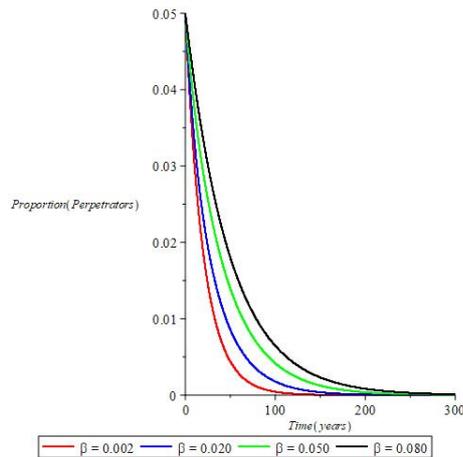Figure. 4. Simulations of $v(t)$ with multiple initial conditions

Figure. 5. Dynamics of the perpetrator proportion under different values of β

zero over the time interval of 300 years, indicating the eventual elimination of perpetrators in the long term. For smaller values of β, the reduction in the perpetrator proportion proceeds more rapidly, while larger values of β result in a more gradual decrease, allowing the perpetrator population to persist for a longer duration before approaching extinction. This behavior demonstrates that β plays a crucial role in determining the speed of convergence toward the equilibrium, without altering the stability of the final state. In summary, higher values of β slow down the decline of $v(t)$, whereas lower values accelerate the system's progression toward elimination, although all parameter settings ultimately converge to the same long-term outcome.

The results suggest that a high recidivism rate (β) can significantly decelerate the reduction of active perpetrators, indicating that punitive measures alone are in-

sufficient without complementary efforts to prevent reoffending. Within the $SVPR$ framework, the transition from the punished compartment back to perpetrators reflects systemic shortcomings in rehabilitation, post-sentence supervision, counseling access, social reintegration, and the mitigation of underlying risk factors. Consequently, policy measures such as strengthened offender rehabilitation programs, community-based monitoring, continuous education, risk-based supervision or offender registries, and coordinated interagency interventions are essential for effectively reducing the value of β. These insights underscore that lowering recidivism not only accelerates the decline of the perpetrator population but also enhances the long-term success of prevention and intervention strategies.

5. Conclusion

The deterministic model employed in this study is utilized to analyze the dynamics of sexual violence. A comprehensive investigation was conducted to ascertain the fundamental parameters, which included the basic reproduction number (R_0), the sexual violence-free equilibrium (E_0), and the endemic equilibrium (E_*). Through meticulous analysis and rigorous calculation, these critical parameters were determined, providing a foundation for further study and analysis in this field. The findings indicate that under specific conditions, both equilibrium points exhibit local asymptotic stability.

This study identifies the parameters α and β as critical drivers in the spread of sexual violence. Parameter α denotes the effective contact rate between susceptible individuals and perpetrators, while parameter β represents the transition rate from punished to being a perpetrator once more. The reduction of both parameters is an essential strategy for the effective curtailment of sexual violence within the population. However, the current model does not account for the influence of prevention and rehabilitation interventions, particularly in reducing recidivism among punished individuals. Therefore, future research could incorporate optimal control interventions to better represent realistic strategies for mitigating sexual violence.

Bibliography

[1] World Health Organization (WHO), 2024, *Violence information.* Available from: https://apps.who.int/violence-info/sexual-violence, accessed 8 May 2025

[2] WHO, Violence Against Women Prevalence Estimates 2018, Available from https://iris.who.int/server/api/core/bitstreams/145c0488-87d1-4d3f-a562-f98f9bd5dac2/content

[3] Ngabito, S.M.L., Wahiduddin, I.L.M. and Amiruddin, R., 2024. The Effect of Educational Media Website and Surveillance on Risk Behavior for Prevention of Premarital Sex and Sexual Violence in Adolescents in Gorontalo Regency High School, *International Journal of Statistics in Medical Research* Vol. **13**: 238 – 244

[4] Saini, M., Kaur, M., Sengupta, E. and Ahmed, K., 2023, Artificial intelligence inspired framework for preventing sexual violence at public toilets of educational institutions with the improvisation of gender recognition from gait sequences. *Soft Computing* Vol. **27**(13): 8739 – 8758

[5] Gupta, A. and Nazrin, C.S., 2021, The main factors of intimate partner violence-a statistical study. *Journal of Statistics Applications & Probability* Vol. **10**: 103 – 112

[6] Leal-Enríquez, E. and Gutiérrez-Antúnez, A.R., 2020, Indicators of Violence Levels: Questionnaires and Predictive Mathematical Model, *Modelling and Simulation in Engineering* Vol. **2020**(1): 5857263

[7] Bahri, S., Riyandi, H.R. and Rudianto, B., 2024. Stability of the Dynamic Model of SVPR Sexual Violence Cases. *BAREKENG: Jurnal Ilmu Matematika dan Terapan* Vol. **18**(3): 1719 – 1728

[8] Maharani, M., Ismaya, S., Paulina, A. L., Siagian, M. N., Tontowi Puteri, B. P., et al., 2025, *Research Summary: Sentencing Disparity in Sexual Assault and Rape Cases in Indonesia 2019–2021*, Indonesia Judicial Research Society (IJRS)

[9] Gholami, H., Gachpazan, M. and Erfanian, M., 2025,A three-layered compartmental model of Nipah virus transmission with analysis. *Journal of Mathematical Modeling* Vol. **13**(2): 445 – 466

[10] Agbomola, J.O. and Loyinmi, A.C., 2022, Modelling the impact of some control strategies on the transmission dynamics of Ebola virus in human-bat population: An optimal control analysis, *Heliyon* Vol. **8**(12): e12121

[11] Fahimi, M., Nouri, K. and Torkzadeh, L., 2025, Analytical investigation of fractional SEIRVQD measles mathematical model, *Journal of Mathematical Modeling* Vol. **13**(2): 393 – 413

[12] Delamater, P.L., Street, E.J., Leslie, T.F., Yang, Y.T. and Jacobsen, K.H., 2019, Complexity of the basic reproduction number (R0), *Emerging Infectious Diseases* Vol. **25**(1): 1 – 4

[13] Borah, P.B., Dehingia, K., Sarmah, H.K. and Emadifar, H., 2025, Dynamics of simultaneous propagation of two COVID-19 strains, *Advances in Continuous and Discrete Models* Vol. **2025**(1): 45

[14] Fisman, D.N., Greer, A.L. and Tuite, A.R., 2020, Bidirectional impact of imperfect mask use on reproduction number of COVID-19: A next generation matrix approach. *Infectious Disease Modelling* Vol. **5**: 405 – 408

[15] Trachoo, K., Chaiya, I. and Prathumwan, D., 2025, An improved mathematical modeling of bone remodeling: the role of stability in predicting bone health. *Advances in Continuous and Discrete Models* Vol. **2025**(1): 73

[16] Fisher, S.D., 1999, *Complex variables*, Courier Corporation

[17] Hao, W., Zhang, J., Peng, Z. and Jin, Z., 2025, Evaluating the current status and strategies of achieving the 95-95-95 target for HIV/AIDS: A mathematical model. *Discrete and Continuous Dynamical Systems-B* Vol. **30** Issue 10: 3829 – 3857