Jurnal Matematika UNAND
Vol. **15** No. **3** Hal. 319 – 324
Edisi Juli 2026
ISSN : 2303–291X
e-ISSN : 2721–9410

https://doi.org/10.25077/jmua.15.3.319-324.2026

DISTANCE ANTIMAGIC LABELING FOR COPIES OF GRAPH

PETER JOHN,* BONARDO LUBIS, KIKI ARIYANTI SUGENG
Department of Mathematics,
Faculty of Mathematics and Natural Sciences, Universitas Indonesia,
email : peter.john@sci.ui.ac.id, bonardo.lubis@ui.ac.id, kiki@sci.ui.ac.id

Received February 28, 2026 Received in revised form April 30, 2026
Accepted May 7, 2026 Available online July 31, 2026

Abstract. Let G be a graph with vertex set $V(G)$ and edge set $E(G)$. Let f be a bijective function from the vertex set $V(G)$ to the set $\{1, 2, 3, \cdots, |V(G)|\}$ and weight of vertex v in $V(G)$ is the sum of labels of all neighbors of vertex v. If there is no pair of vertices of $V(G)$ have equal weight, then f is called a distance antimagic labeling and G is called a distance antimagic graphs. Copies of graph G, denoted as nG is disjoint union of n graphs which each is isomorphic to G. In this paper, we present several results on distance antimagic labeling of copies of graph.

Keywords: copies of graph, distance antimagic labeling, distance antimagic graphs

1. Introduction

In [1], distance antimagic labeling of graph $G = (V, E)$ introduced by Kamachi and Arumugam as a bijective function $f : V(G) \to \{1, 2, \cdots, |V(G)|\}$ such that for any pair vertices $u, v \in V(G)$ satisfy $\sum_{x \in N(u)} f(x) \neq \sum_{y \in N(v)} f(y)$, where $N(u)$ is the set of all neighbor of vertex u. Any graph G is called a distance antimagic graph if there exists a distance antimagic labeling on G. They proved that Petersen graph, cycle C_n with $n \neq 4$, wheel W_n with $n \neq 4$, and path P_n are distance antimagic graph. They made a conjecture which stated "a graph G is a distance antimagic if and only if $N(u) \neq N(v)$ for any two distinct vertices $u, v \in V(G)$".

Handa, Godinho, and Singh [2] showed that generalised Petersen graph $P(n, k), n \geq 5$ and Harary graph $H_{4,n}, n \neq 6$ are distance antimagic graphs. Handa et al. [3] gave a tool for proving the existence of distance antimagic labelings for join and corona of two graphs.

Simanjuntak et al. [4] showed that disjoint union of two path $P_n \cup P_m$, with $n, m \geq 3$, copies of path P_n with $n \neq 3$, copies of wheel mW_n with $n \geq 3$, copies of

*Corresponding author

fan mF_n, copies of friendship mf_n with $n \geq 3$, and copies of star mS_n are distance antimagic graph. they also proposed few open problems, one of them is "If G is a non-regular graph containing no two vertices having the same neighborhood set, show that nG is distance antimagic".

In 2022, Simanjuntak and Tritama [5] proved that some product graph are distance antimagic graphs. Those product graph are cartesian product of $P_n \square K_2$ when $n \neq 2$, $K_{n,n} \square K_2$ when $n \neq 1$, and $P_n \square K_2$. They also proved that if G is a distance antimagic regular graph, then the corona product $K_1 \bigodot G$ is a distance antimagic graph.

In this paper, we give partial proof of problem given in [4].

2. Result and Discussion

In this section, we begin the discussion from distance antimagic labeling of disjoint union of two graphs. In [4], Simanjuntak et al. proved that two graphs G and H with certain properties will have distance antimagic labeling in the following theorem.

Theorem 2.1. [4] *Let G and H be two distance antimagic graphs. If H is r-regular and $N(u) \leq r$ for every $u \in V(G)$ then $G \cup H$ is also distance antimagic graph.*

Next, we show that copies of regular and distance antimagic graph, Herschel graph, and copies of Herschel graph are distance antimagic graph. In [6], Burr et al stated as follows. Let $G = (V, E)$ be a graph. For any positive integer n, n copies of graph G, denoted as nG is disjoint union of G_i, where $G_i \cong G, i = 1, 2, \cdots, n$.

Theorem 2.1 can be use to prove copies of graph is distance antimagic graph, when the graph is regular and distance antimagic graph. It states in the following corollary.

Corollary 2.2. *Let graph G be a r-regular graph. If G is a distance antimagic graph, then the n copies of graph G, nG, is a distance antimagic graph.*

Proof. Let G be an r-regular and distance antimagic graph. Let $P(n)$ be the proposition that the disjoint union of n copies of G is a distance antimagic graph. We use mathematical induction to prove that $P(n)$ is true for all $n \geq 2$.

For the base case $n = 2$, since $2G = G \cup G$, it follows from Theorem 2.1 that $2G$ is a distance antimagic graph. Next, assume that $P(k)$ is true for some arbitrary positive integer $k \geq 2$. Since G is r-regular, the graph kG is also r-regular. Then, by Theorem 2.1, we conclude that $kG \cup G$ is a distance antimagic graph. By the definition of disjoint union of copies, we have $kG \cup G = (k+1)G$. Hence, $P(k+1)$ is true under the assumption that $P(k)$ is true. Therefore, by mathematical induction, we have proven that if G is an r-regular and distance antimagic graph, then any finite disjoint union of copies of G is also a distance antimagic graph. $\square$

Similar contruction in Theorem 2.1 cannot implied directly to copies of non regular graphs. As an example, we can see the case for path. Let graph P_6 with $V(P_6) = \{v_1, v_2, v_3, v_4, v_5, v_6\}$ and $E(P_5) = \{v_1v_2, v_2v_3, v_3v_4, v_4v_5, v_5v_6\}$. Define a function $f : V(P_6) \to \{1, 2, \cdots, 6\}$ with $f(v_i) = i$, for $i = 1, 2, \cdots, 6$. Since f is an increasing function and $|V(P_6)| = 6$ then f is bijective. Hence, we get $w(v_1) = 2$, $w(v_2) = 4$, $w(v_3) = 6$, $w(v_4) = 8$, $w(v_5) = 10$, and $w(v_5) = 5$. Therefore, graph P_6 is distance antimagic graph. Figure 1 shows the vertices' label of graph $2P_6$ when we use similar construction from Theorem 2.2. Then two vertices which are vertex with label 4 and vertex with label 7 have same weight, which is 8. Hence the function that we constructed is not distance antimagic labeling.

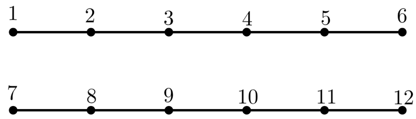

Figure. 1. Graph $2P_6$.

However, Simanjuntak et al. [4] proved that the union of two path graphs is a distance antimagic graph, as stated in the following theorem.

Theorem 2.3. [*4*] *For any positive integers $m, n > 3$, the disjoint union of two paths $P_m \cup P_n$ is a distance antimagic graph.*

In the rest of this section, we will give distance antimagic labeling for Herschel graph and distance antimagic labeling construction for copies of Herschel graph. In [7] and [8], the Herschel graph H is defined as a bipartite graph with order 11 and size 18 which is the smallest non hamiltonian polyhedra graph, as in Figure 2.

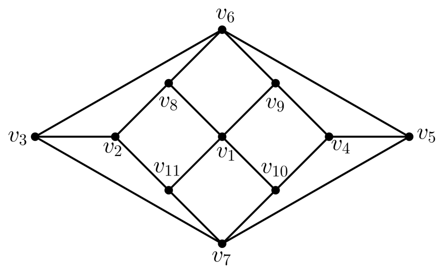

Figure. 2. Herschel graph.

Theorem 2.4. *Herschel graph is a distance antimagic graph.*

Proof. Let H be graph Herschel with vertex name shown in Figure 3.

We define the bijection $f : V(H) \to \{1, 2, \cdots, 11\}$ as $f(v_i) = i, i = 1, 2, \cdots, 11$ as in Figure 4.

Hence, the weights of all vertices of H are shown in Figure 4. Since all vertices have different weights, we conclude that Herschel graph is a distance antimagic graph. $\square$

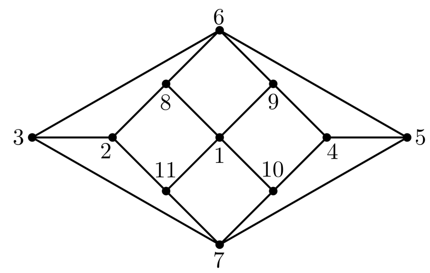

Figure. 3. Vertex label on Herschel graph.

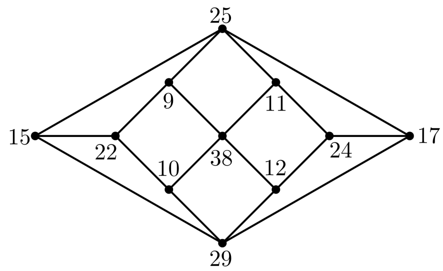

Figure. 4. Vertex weight on Herschel graph.

Next, we prove that copies of Herschel graph is a distance antimagic graph.

Theorem 2.5. *Copies of Herschel graph nH with $n \geq 2$, is a distance antimagic graph.*

Proof. Let nH be the copies of Herschel graph H and H_j be the j-th component of nH. Figure 5 shows the name of all vertices belong to $H_j, j = 1, 2, \cdots, n$.

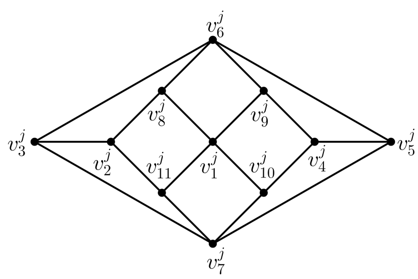

Figure. 5. Component H_j of graph nH.

Let f be the function defined in Theorem 2.4. We define $g : V(nH) \to \{1, 2, \cdots, 11n\}$ as $g(v_i^j) = f(v_i) + 11(j-1)$ for $i = 1, 2, \cdots, 11$, and $j = 1, 2, \cdots, n$.

For each $i \in \{1, 2, \cdots, 11\}$, the sequence $(g(v_i^j))$ is an arithmetic sequence with common different 11 and $g(v_i^j) \equiv i(\text{mod } 11)$. Hence, no vertices have same label. Since the smallest label of $g(v_i^j)$ is 1 when $i = j = 1$, and the largest label of $g(v_i^j)$ is $11n$ when $i = 11$ and $j = n$, then g is bijective. Next, we calculate the weight of vertices of nH, and we get:

$$\begin{aligned} w(v_1^j) &= 44(j-1) + 38, \\ w(v_2^j) &= 33(j-1) + 22, \\ w(v_3^j) &= 33(j-1) + 15, \\ w(v_4^j) &= 33(j-1) + 24, \\ w(v_5^j) &= 33(j-1) + 17, \\ w(v_6^j) &= 44(j-1) + 25, \\ w(v_7^j) &= 44(j-1) + 29, \\ w(v_8^j) &= 33(j-1) + 9, \\ w(v_9^j) &= 33(j-1) + 11, \\ w(v_10^j) &= 33(j-1) + 12, \\ w(v_11^j) &= 33(j-1) + 10, \end{aligned}$$

for $j = 1, 2, \cdots, n$.

For each $i \in \{1, 2, \cdots, 11\}$, the sequence $(w(v_i^j))$ is an increasing arithmetic sequence, then $w(v_i^{j_1}) \neq w(v_i^{j_2})$ when $j_1 \neq j_2$. Let $i_1 \neq i_2$, the residue modulo 11 of any terms in sequence $(w(v_{i_1}^j))$ is not equal with the residue modulo 11 of any terms in sequence $(w(v_{i_2}^j))$, except when $i_1 = 2$ and $i_2 = 9$. Every term in the sequence $(w(v_2^j))$ has different residue modulo 33 compares to every term in the sequence $(w(v_9^j))$. So, for any $u, v \in V(nH)$ and $u \neq v$, we have $w(u) \neq w(v)$. Hence, we conclude that f is distance antimagic labeling. In other word, n copies of Herschel graph is a distance antimagic graph for any positive integer $n \geq 2$. $\square$

Figure 6 shows the vertices' label of 4 copies of Herschel graph from the distance antimagic labeling given in the proof of Theorem 2.5.

3. Conclusion

In this paper we show that for any regular and distance antimagic graph then the copies of the graph also distance antimagic graph. Moreover, we also show that for Herschel graph and the copies of Herschel graph also distance antimagic. For the future research, we can find whether the copies of distance antimagic non regular graphs is also distance antimagic.

Bibliography

[1] Kamachi, N. and Arumugam, S, 2013, Distance Antimagic Graphs, *Journal of Combinatorial Mathematics and Combinatorial Computing*, Vol. **84**: 61 – 67

[2] Handa, A. K., Godinho, A., Singh, T., 2016, Some Distance Antimagic Labeled Graphs, *Algorithms and Discrete Applied Mathematics*, Vol. **9602**: 190 – 200

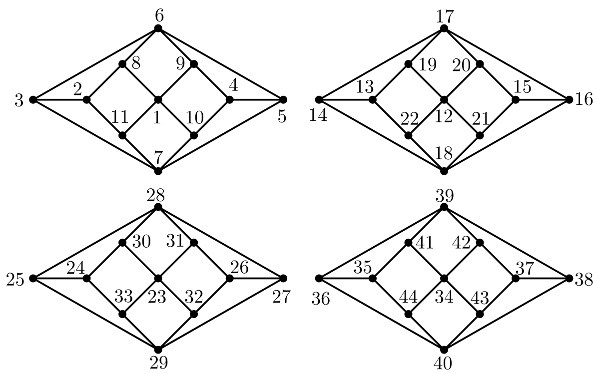

Figure. 6. Vertex label on 4 copies of Herschel graph.

[3] Handa, A. K., Godinho, A., Singh, T., Arumugam, S., 2017, Distance antimagic labeling of join and corona of two graphs, *AKCE International Journal of Graphs and Combinatorics*, Vol. **14**(2): 172 – 177

[4] Simanjuntak, R., Nadeak, T., Yasin, F., Wijaya, K., Hinding, N., Sugeng, K. A., 2021, Another Antimagic Conjecture, *Symmetry*, Vol. **13**: 2071

[5] Simanjuntak, R. and Tritama, A. 2022, Distance Antimagic Product Graphs, *Symmetry*, Vol. **14**: 1411

[6] Burr, A., Erdős, P., Spencer, J.H., 1975, Ramsey theorems for multiple copies of graphs, *Transactions of The American Mathematical Society*, Vol. **209**: 87 – 99

[7] Gallian, J. A., 2024, A dynamic survey of graph labeling, *The Electronic Journal of Combinatorics*, Vol. **17**

[8] West, D. B., 2001, *Introduction to Graph Theory*, Prentice Hall, 2nd edition