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# ON N-SOFT HYPERGRAPH

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**Abstract**. In this paper we explore the notion of N-soft hypergraphs, which is an hybridization of N-soft set and hypergraph. Next, we define strong N-soft hypergraphs, and induced complete N-soft hypergraphs. We present the definitions of some of their unary operations and examine their properties. Then, we give illustration on how N-soft hypergraph will be applied in portray real-life problem.

*Keywords*: Soft Graph; N-Soft Hypergraph; N-Soft Sets

## 1. Introduction

Many models have been developed to handle uncertainty. Binary models often fail to capture the nuanced relationships present in real-world cases. To handle these limitations, concepts like degree-of-truth or parameterized approach could be used.

The notion of fuzzy sets, a method for representing uncertainty and vagueness using degrees of truth, was introduced by Zadeh [1]. The concept of soft sets, which represents uncertainty through a parameterized approach, was introduced by Molodtsov [2]. These two concepts have been applied to graphs, yielding fuzzy graphs [3] and soft graphs [4]. This integration takes advantage of their ability to model vagueness in binary relations.

To further make use of this advantage, the hybridisation can be extended to hypergraphs. Hypergraphs is an extension of graphs where the notion of an edge is generalised such that an edge could connect more than two vertices. Therefore, it is capable of representing multi-ary relations. Fuzzy hypergraph has been introduced since [5]. Many extension of fuzzy hypergraph have been presented in [6]. On the other hand, there have been some extension that involves soft set, such as intuitionistic fuzzy soft hypergraph [7,8] and Phytagorean fuzzy soft hypergraph [9], while soft hypergraph has been introduced later in [10]

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The continuous nature of membership and non-membership degrees, i.e. [0,1] intervals, can be computationally intensive. In another approach, Fatima in [11] introduces the N-soft set, inspired by the rating system. This approach simplifies the computation while maintaining its granularity. Some operations and applications were explored by in [12], and research or development on N-soft set is still progressing [13,14]

## 2. Preliminary Notes

An hypergraph is defined as follow,

**Definition 2.1.** [15] *Let $X$ be a non-empty set and $E$ is a collection of non-empty subsets of $X$. Hypergraph $H$ over $X$ is an ordered pair $H = (X, E)$, where $X$ is called set of vertices and $E$ is called set of edges.*

Two vertices of an hypergraph are called *adjacent* if there exists a hyperedge containing both of these two vertices. Two hyperedges are *incident* if they have a non-empty intersection.

Soft set over a universe $U$ is a set-valued function from set of parameter $E$ to power set of the universe $\mathcal{P}(U)$. Molodtsov defined soft set as follows. Let $U$ be a universe of discourse and $E$ is set of parameters.

**Definition 2.2.** [2] *Ordered pair $(F, E)$ is called soft set (over U) if and only if F is a mapping from E into the set of all subset of U.*

The concept of soft sets was later applied to graph theory, yielded the notion of soft hypergraph, which was introduced by Thumbakara in [10] as follows

**Definition 2.3.** [10] *Let $H^* = (V, E)$ be a simple hypergraph with vertex set $V$ and edge (hyperedge) set $E$ and $C$ be any nonempty set. let $E_s$ be the set of all subhyperedges of $H^*$. Let $R$ be an arbitrary relation between elements of $C$ and elements of $V$. That is $R \subseteq C \times V$. A mapping $A : C \to P(V)$ can be defined as $A(c) = v \in V : cRv$. Also define a mapping $B : C \to P(E_s)$ by $B(c) = m-subhyperedges(A(c))$. The pair $(A, C)$ is a soft set over $V$ and the pair $(B, C)$ is a soft set over $Es$. Then the 4-tuple $H = (H^*, A, B, C)$ is called a Soft Hypergraph if it satisfies the following conditions:*

*(1) $H^* = (V, E)$ is a simple hypergraph,*
*(2) $C$ is a non-empty set of parameter,*
*(3) $(A, C)$ is a soft set over $V$,*
*(4) $(B, C)$ is a soft set over $E_s$,*
*(5) $(A(c), B(c))$ is a semisubhypergraph of $H^*$ for all $c \in C$.*

The notion of soft sets was later extended by assigning rating values to its elements. This new notion is called N-soft set, introduced by Fatima et. al. in [11] as follows.

**Definition 2.4.** [11] *Let $U$ be a universe set of objects and $E$ be attributes, $A \subseteq E$. Let $R = \{0, 1, \cdots, N-1\}$ be a set of ordered grades where $N \in \{2, 3, \cdots\}$. We say* *that $(F, A, N)$ is an $N$-soft set on $U$ if $F : A \to 2^{U \times R}$, with the property that for eacha $\in A$ and there exists a unique $(u, ra) \in U \times R$ such that $(u, ra) \in F(a), u \in U, ra \in R$.*

## 3. N-Soft Hypergraph

Intuitively, N-soft hypergraph is a collection of hypergraphs (one for each parameter) that each vertex and hyperedge are weighted with a natural number (which is called rating) less than N such that the weight of an edges does not exceed the minimum weight of vertices adjacent into it.

**Definition 3.1.** *Let $X = \{x_1, x_2, \cdots, x_n\}$ be a non-empty set, $A = \{\alpha_1, \alpha_2, \cdots\}$ be a non-empty set of parameters, $R = \{0, 1, 2, \cdots, N-1\}$ for $N \in \{2, 3, 4, \cdots\}$. Let $F_X : A \to \mathcal{P}(X)$, and $F_X(\alpha) : X \to R$. Triple $(F_X, A, N)$ is called N-soft set of vertices. Let $E_X : A \to \mathcal{P}(\mathcal{P}(X) \setminus \emptyset)$ and $E_X(\alpha) : (\mathcal{P}(X) \setminus \emptyset) \to R$. Triple $(E_X, A, N)$ is called N-soft set of edges. A pair $((F_X, A, N), (E_X, A, N))$ is called N-soft hypergraph over $X$ iff $E_X(\alpha)(e) \leq \min\{F_X(\alpha)(x) \mid x \in e\}$ for every $\alpha \in A$ and for every $e \in \mathcal{P}(X)$. We can denote a N-soft hypergraph $((F_X, A, N), (E_X, A, N))$ as $(F_X, E_X, A, N)$.*

In drawing an N-soft hypergraph, we represent a vertex by a point, and an hyperedge by a circle enclosing vertices incident to it. The rating of a vertex and an edge written nearby. When listing the ratings of edges, any possible edge that is not included is considered to have a rating of zero for all parameters.

**Example 3.2.** Let $X_1 = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7\}$ and $A = \{\alpha_1, \alpha_2\}$. Consider a N-soft hypergraph $H_1 = (F_{X_1}, E_{X_1}, A, 6)$ as shown in Fig. 1, where the definition of $F_X$ and $E_X$ is given by Table 1. $H_1$ is indeed an N-soft hypergraph since $E_X(\alpha)(e) \leq \min\{F_X(\alpha)(x) | x \in e\}$ for all $\alpha \in A$ and for all $(e) \in \mathcal{P}(X)$.

Table 1. Tabular form of $H_1$

| $F_X$ | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $x_7$ |
|---|---|---|---|---|---|---|---|
| $\alpha_1$ | 3 | 5 | 4 | 5 | 3 | 1 | 3 |
| $\alpha_2$ | 4 | 4 | 4 | 3 | 4 | 2 | 3 |

| $E_X$ | $x_7$ | $x_1x_2$ | $x_2x_3$ | $x_4x_5$ | $x_1x_3x_4$ | $x_4x_5x_6x_7$ |
|---|---|---|---|---|---|---|
| $\alpha_1$ | 3 | 3 | 0 | 3 | 3 | 0 |
| $\alpha_2$ | 0 | 0 | 4 | 0 | 3 | 2 |

Next, we consider N-soft subhypergraph.

**Definition 3.3.** *Let $H_1 = (F_{X_1}, E_{X_1}, A, N)$ is an N-soft hypergraph over $X_1$ and $H_2 = (F_{X_2}, E_{X_2}, B, M)$ is an N-soft hypergraph over $X_2$, then $H_2$ is an N-soft subhypergraph of $H_1$ (or in another word, $H_1$ is a superhypergraph of $H_2$) if $X_2 \subseteq X_1$, $B \subseteq A$, and $F_{X_1}(\alpha)(x) = F_{X_2}(\alpha)(x)$, $E_{X_1}(\alpha)(e) = E_{X_2}(\alpha)(e)$ for all $\alpha \in B$, $x \in X_2$, $e \in \mathcal{P}(X_2)$, and $M \leq N$. If $H_1 \subseteq H_2$ and $H_2 \subseteq H_1$ then we say $H_1 = H_2$*

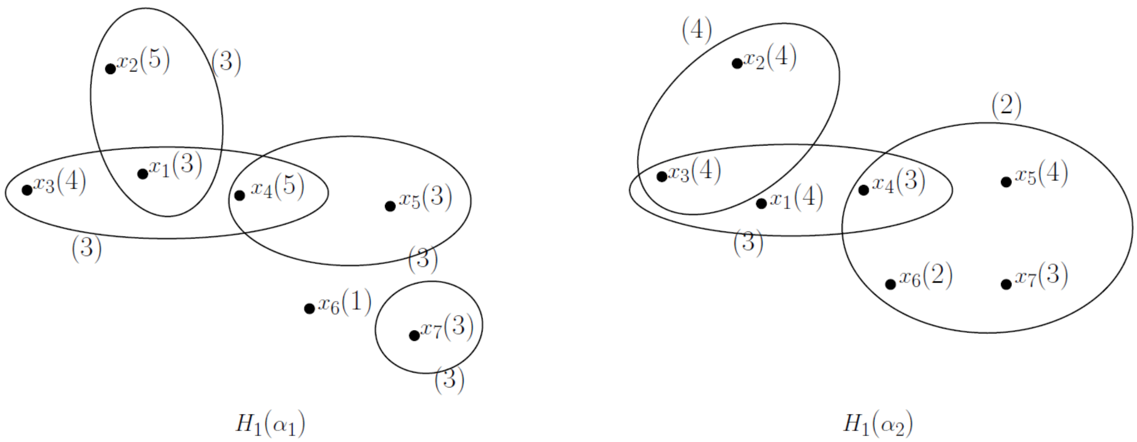


Figure. 1. Hypergraph $H_1 = (F_X, E_X, A, 6)$

Table 2. Tabular form of $H_2$

| $F_W$ | $x_1$ | $x_2$ | $x_3$ | $E_W$ | $x_1x_2$ | $x_2x_3$ |
|---|---|---|---|---|---|---|
| $\alpha_1$ | 3 | 5 | 4 | $\alpha_1$ | 3 | 0 |
| $\alpha_2$ | 4 | 4 | 4 | $\alpha_2$ | 0 | 4 |


Figure. 2. Hypergraph $H_2 = (X_W, E_W, A, 5)$

**Example 3.4.** Consider N-soft hypergraph $H_1$ from Example 3.2 and $H_2$ N-soft hypergraph over $W = \{x_1, x_2, x_3\}$ where $F_W$ and $E_W$ given in Table 2.

It is clear that $W \subseteq X$, and $F_X(e)(x) = F_W(e)(x)$, $E_X(e)(e) = E_W(e)(e)$ for all $e \in W$, $e \in \mathcal{P}(W)$, and $M \leq N$. Therefore, $H_2$ is an N-soft subgraph of $H_1$.

We previously defined an N-soft subhypergraph to have its set of parameters as a subset of the parameters of its superhypergraph. This allows us to consider a subhypergraph associated with a specific single parameter to be an N-soft hypergraph of its own.

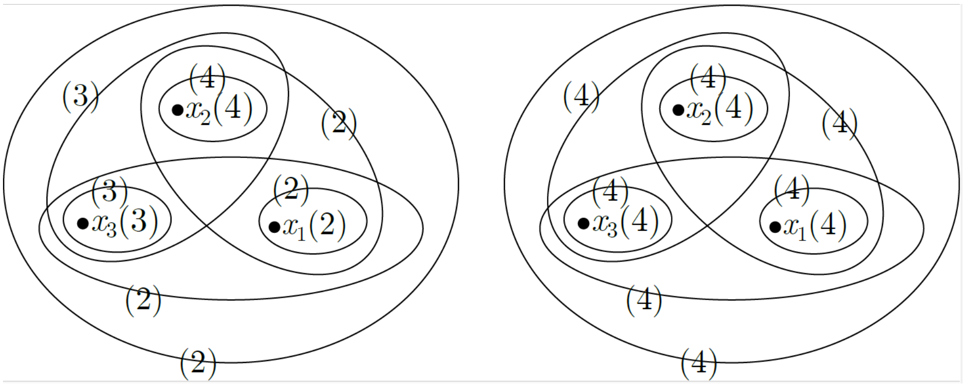


Figure. 3. Hypergraph $K(F_W)$

**Definition 3.5.** *Let $H = (F_X, E_X, A, N)$, and $\alpha_1 \in A$, then $H(\alpha_1) = (F_X, E_X, \{\alpha_1\}, N)$ is subhypergraph of $H$ corresponding to parameter $\alpha_1$*

Notice that the grade of vertices are arbitrary, but the grade of each hyperedge is restricted by some condition depending on the vertices incides to it. The maximum possible grade for an edge, called *strength*, is the minimum grade of vertices incides to it, denoted as $\overline{E_X}(\alpha) = \min\{F_X(x) \mid x \in e\}$. Any vertices or edges that are not graded are considered to have a grade of zero.

**Definition 3.6.** *Let $H = (F_X, E_X, A, N)$ is an N-soft hypergraph over $X$. N-soft hypergraph $H$ is called a strong N-soft hypergraph if $E_X(\alpha)(e) = \overline{E_X}(\alpha)(e)$ or $E_X(\alpha)(e) = 0$ for all $e \in \mathcal{P}(X) \setminus \emptyset$ and for all $\alpha \in A$.*

With this definition, we can say that $H_2$ is a strong N-soft hypergraph, while $H_1$ is not a strong N-soft hypergraph. If we inspect further to each subhypergraph that corresponding to a parameter,$H_1(\alpha_1), H_2(\alpha_1), H_2(\alpha_2)$ is a strong N-soft hypergraph, while $H_1(\alpha_2)$ is not a strong N-soft hypergraph.

**Definition 3.7.** *Let $F_X$ is a vertices rating function over $X$, then N-soft hypergraph $H = (F_X, E_X, A, N)$ over $X$ is called complete hypergraph induced by $F_X$, notated $K(F_X)$, if $E_X(\alpha)(e) = \overline{E_X}(\alpha)(e)$ for all $e \in \mathcal{P}(X) \setminus \emptyset$ and for all $\alpha \in A$.*

**Example 3.8.** Complete N-soft hypergraph induced by $F_W$ from Example 3.2 is N-soft hypergraph $K(F_W)$ which is given in Figure 3.

**Definition 3.9.** *Let $F_X$ is a vertices rating function over $X$. Then k-uniform N-soft hypergraph induced by $F_X$ is $E_k(F_X) = (F_X, E_X, A, N)$ where $E_X(\alpha)(e) = \overline{E_X}(\alpha)(e)$ for all $e$ which have cardinality $k$ and for all $\alpha \in A$.*

**Definition 3.10.** *Let $H = (F_X, E_X, A, N)$. Underlying crisp hypergraph of $H$ is $H^* = (V^*, E^*)$ where:*
*$V^* = \{x \in X | F_X(\alpha)(x) > 0$ for some $\alpha \in A\}$,*
*$E^* = \{e \in \mathcal{P} \setminus \emptyset | E_X(\alpha)(e) > 0$ for some $\alpha \in A\}$.*

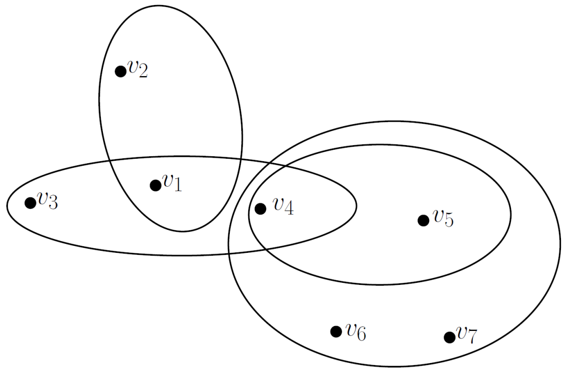



Figure. 4. Hypergraph $H^* = (V^*, E^*)$

Table 3. Tabular form of $H_2$

| $F_W$ | $x_1$ | $x_2$ | $x_3$ | $E^c_W$ | $x_1$ | $x_2$ | $x_2$ | $x_1x_2$ | $x_1x_3$ | $x_2x_3$ | $x_1x_2x_3$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\alpha_1$ | 3 | 5 | 4 | $\alpha_1$ | 3 | 5 | 4 | 0 | 3 | 4 | 3 |
| $\alpha_2$ | 4 | 4 | 4 | $\alpha_2$ | 4 | 4 | 4 | 4 | 4 | 0 | 4 |

**Example 3.11.** The underlying crisp hypergraph of a N-soft hypergraph $H_1$ from Example 3.2 is hypergraph shown in Figure. 4

Following Demir [12] who defines complement of a N-soft set, we define complement of a N-soft hypergraph as follows.

**Definition 3.12.** *Complement of N-soft hypergraph $H = (F_X, E_X, A, N)$ over $X$ denoted by $H^c$ and defined as $H^c = (F_X, E^c_X, A, N)$, where:*

$$E^c_X(\alpha)(e) = \overline{E_X}(\alpha)(e) - E_X(\alpha)(e),$$

*for all $e \in \mathcal{P}(X)$.*

**Example 3.13.** Consider $H_2$ from Example 3.4. Rating of vertices and edges of $H^c_2$ are defined in Table 3.

**Theorem 3.14.** *If $H$ is an N-soft hypergraph, then $H^c$ is an N-soft hypergraph. Furthermore, if $H$ is a strong N-soft hypergraph, then $H^c$ is a strong N-soft hypergraph.*

**Proof.** Notice that all edges in $H^c$ satisfy the requirement defined in Definition 3.1, that is:

$$E^c_X(\alpha)(e) = \overline{E_X}(\alpha)(e) - E_X(\alpha)(e) \leq \overline{E_X}(\alpha)(e) = \min\{F_X(\alpha)(x) \mid x \in e\}.$$

Furthermore, if $H$ is a strong N-soft hypergraph, then $E_X(\alpha)(e) = \overline{E_X}(\alpha)(e)$ or $E_X(\alpha)(e) = 0$. Consequently, $E^c_X(\alpha)(e) = \overline{E_X}(\alpha)(e) - \overline{E_X}(\alpha)(e) = 0$, or $E^c_X(\alpha)(e) = \overline{E_X}(\alpha)(e) - 0 = \overline{E_X}(\alpha)(e)$. This makes $H^c$ also a strong N-soft hypergraph. $\square$

**Theorem 3.15.** *If $H$ is an N-soft hypergraph, then $(H^c)^c = H$.*

**Proof.** Notice that when taking complement, the rating of vertices are not changing. Thus, for all edges in $H$, $\overline{E^c_X}(\alpha)(e) = \overline{E_X}(\alpha)(e)$. Furthermore,

$$\begin{aligned}(E^c_X)^c(\alpha)(e) &= \overline{E^c_X}(\alpha)(e) - E^c_X(\alpha)(e),\\ &= \overline{E_X}(\alpha)(e) - \big(\overline{E_X}(\alpha)(e) - E_X(\alpha)(e)\big),\\ &= E_X(\alpha)(e).\end{aligned}$$ □

## 4. Application

N-soft Hypergraph can be applied to represent the assessment of team performance in the workplace. Let us examine the teamwork of some employees based on some parameter, that is, communication, coordination, and reliability. Let the vertex represent an employee, and vertex incident with an hyperedge will interpreted as these employee once work as group. Rating on vertex denotes the score given to a certain employee, while degree on an edge represents their strength as a team. Restriction in N-soft hypergraph could seen as a "collective punishment" assigned into the team for incompetence of one of their members.

For example, assume there are seven employees, denoted by $a, b, c, d, e, f, g$. In previous work, they have been work together in some team, namely $E_1, E_2, E_3, E_4$. The group are shown in Table 4.

Table 4. Table of teams and its members

| Team | Member |
|---|---|
| $T_1$ | $a, b, c, d, e, f, g$ |
| $T_2$ | $a, b, c$ |
| $T_3$ | $a, d, g$ |
| $T_4$ | $g, d, e, f$ |

We examine their teamwork with three parameters, that is, $\alpha_1$ denoting communication, $\alpha_2$ denoting coordination, and $\alpha_3$ denoting reliability. The examination is given in rating from 0 to 10. For an individual, if someone gets a rating of 0, it means they do not appear in the group; a rating of 1 means there are rarely communicate and most of the teams are unaware of what they convey; a rating of 2 means there will be slow communication and others do not understand them. The criterion gradually increases up to a rating of 10, indicating good ability and the capacity to be an example to others. For a team, if a team got rated of 0, that means the group is not doing anything; a rating of 1 means that team members are not focused and miss the deadlines; a rating of 2 some members don't follow through. This criterion gradually rise until rating of 10, which means they have done perfect outcomes and always can be trusted.

Let the examination runs and the results are in the Table 5 and Table 6. Then, the table could be illustrated by N-soft hypergraph $H_3 = (F_B, E_B, A, 11)$ for $B = \{a, b, c, d, e, f, g\}$, and $A = \{\alpha_1, \alpha_2, \alpha_3\}$ as shown in Figure 5.

Table 5. Tabular Form of personal assesment.

| $F_B$ | a | b | c | d | e | f | g |
|---|---|---|---|---|---|---|---|
| $\alpha_1$ | 7 | 5 | 6 | 7 | 6 | 3 | 8 |
| $\alpha_2$ | 8 | 5 | 7 | 8 | 4 | 6 | 6 |
| $\alpha_3$ | 6 | 5 | 7 | 9 | 5 | 4 | 7 |

Table 6. Tabular Form of teamwork assesment.

| $E_B$ | $T_1$ | $T_2$ | $T_3$ | $T_4$ |
|---|---|---|---|---|
| $\alpha_1$ | 2 | 5 | 6 | 1 |
| $\alpha_2$ | 4 | 5 | 6 | 4 |
| $\alpha_3$ | 3 | 5 | 6 | 4 |

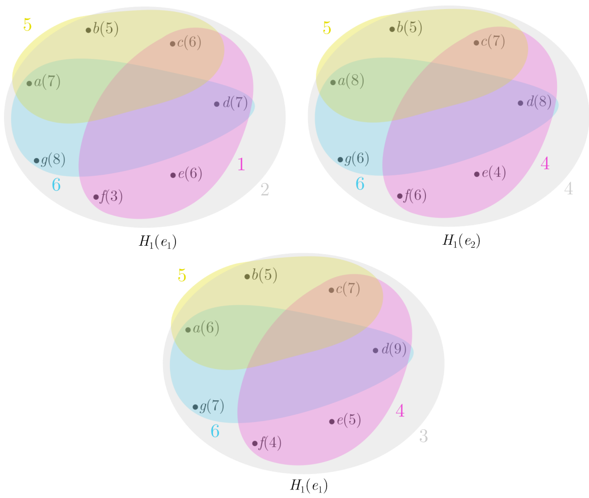


Figure. 5. N-soft hypergraph respresntation of teamwork assesment

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