

CONNECTIVITY INDICES OF POWER GRAPH OVER DIHEDRAL GROUPS D_{2pq}

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Received April 14, 2026 Received in revised form August 5, 2026
 Accepted August 28, 2026 Available online October 22, 2026

Abstract. *The dihedral group, fundamentally characterized by its rotational and reflectional symmetries, serves as an intriguing object of study when represented as a graph. Recently, the study of power graphs and their topological indices has gained significant attention in algebraic graph theory. In this study, we investigate the structural properties of the undirected power graph over the dihedral group D_{2n} specifically for the composite order $n = pq$, where p and q are distinct odd primes. By determining the precise vertex degrees and connectivity patterns within the graph, this paper mathematically derives the exact formulas for five prominent connectivity indices: the first Zagreb index, the Wiener index, the hyper-Wiener index, the Harary index, and the Forgotten index. The results provide a deeper understanding of how the interplay between symmetry elements and the product of two distinct primes influences the topological descriptors of the associated power graph.*

Keywords: Power graph, Dihedral group, Connectivity indices.

1. Introduction

The representation of groups through graphs serves as a powerful approach to understanding the intrinsic properties of groups by analyzing their connectivity patterns. Among various group-graph representations, the power graph is one of the most widely studied. The power graph of a finite group G , denoted by $\Gamma(G)$, was first introduced by Kelarev and Quinn [1]. By definition, two vertices $u, v \in G$ are adjacent if $v = u^k$ or $u = v^k$ for some natural number k . Initially conceptualized as a directed graph [2], the concept was later expanded to the undirected power graph through the seminal work of Chakrabarty et al. [3]. Recent research has continuously explored the properties of these graphs, including their applications to semigroups and the classification of groups based on their power graph structures [4,5].

Current developments in the study of power graphs include the analysis of graph connectivity and diameter in specific groups [6], as well as in-depth investigations into torsion-free groups [7]. Furthermore, the spectra and energy of power graphs have emerged as prominent research topics in recent years [8], demonstrating the profound relevance of this algebraic structure within spectral graph theory.

Parallel to these mathematical advances, connectivity indices in chemical graph theory have become essential tools as molecular descriptors for chemical compounds. Classical indices—such as the hyper-Wiener, Harary, first Zagreb, second Zagreb, and Szeged indices [9,10] have been refined into novel variants like the Sombor and Forgotten indices [11,12]. The application of connectivity indices to group-derived graphs, such as coprime graphs and power graphs, has been actively examined for dihedral and other finite groups [13]. More recently, the connectivity indices of coprime graphs over generalized quaternion groups with more complex orders were discussed in [14].

Despite these extensive studies, research specifically addressing the connectivity indices of power graphs over dihedral groups has been limited to particular orders. Previous work successfully evaluated the connectivity indices of power graphs over dihedral groups for the order $n = p^k$, where p is a prime number [15]. To bridge this existing research gap, this study extends the investigation by modifying the group order to $n = pq$, where p and q are distinct odd primes. The novelty of this research lies in its systematic approach: it begins by determining the structural patterns of the power graph over the dihedral group of order pq . Utilizing these newly established structural properties, the study subsequently derives five prominent connectivity indices: the first Zagreb index, the Wiener index, the hyper-Wiener index, the Harary index, and the Forgotten index.

2. Some Concepts

Throughout this paper, we adopt the notations and graph terminology presented in [16]. To make this article self-contained, the fundamental definitions required for our main results are introduced below.

Definition 2.1. [17] *The dihedral group, denoted by D_{2n} , is given by the presentation:*

$$D_{2n} = \langle a, b \mid a^n = e, b^2 = e, b^{-1}ab = a^{-1} \rangle.$$

Theorem 2.2. [18] *Let $a^i b^j$ be an element of the dihedral group D_{2n} . Then the order of $a^i b^j$ is:*

$$o(a^i b^j) = \begin{cases} \frac{n}{\gcd(i, n)}, & \text{if } j = 0, \\ 2, & \text{if } j = 1. \end{cases}$$

The definitions of the following six graph indices are given below.

Definition 2.3. [19] *Let Γ be a simple connected graph. The First Zagreb index of*

Γ , denoted by $M_1(\Gamma)$, is defined as:

$$M_1(\Gamma) = \sum_{v \in V(\Gamma)} \deg(v)^2,$$

where $\deg(v)$ denotes the degree of the vertex v , i.e., the number of edges incident to v .

Definition 2.4. [20] Let Γ be a simple connected graph. The Wiener index of Γ , denoted by $W(\Gamma)$, is defined as:

$$W(\Gamma) = \sum_{u, v \in V(\Gamma)} d(u, v),$$

where $d(u, v)$ represents the distance between vertices u and v , namely the length of the shortest path connecting them.

Theorem 2.5. [20] Let Γ be a simple connected graph with vertex set $V(\Gamma)$ and edge set $E(\Gamma)$. If the distance between any two distinct vertices of Γ is at most 2, then the Wiener index of Γ satisfies:

$$W(\Gamma) = 2 \binom{|V(\Gamma)|}{2} - |E(\Gamma)|.$$

Definition 2.6. [21] Let Γ be a simple connected graph. The hyper-Wiener index of Γ , denoted by $WW(\Gamma)$, is defined as:

$$WW(\Gamma) = \frac{1}{2} \left(W(\Gamma) + \sum_{u, v \in V(\Gamma)} d(u, v)^2 \right),$$

where $d(u, v)$ is the distance between vertices u and v .

Theorem 2.7. [21] Let Γ be a simple connected graph with $|V(\Gamma)| = n$ and diameter $\text{diam}(\Gamma) \leq 2$. Then the hyper-Wiener index of Γ is given by:

$$WW(\Gamma) = 3 \binom{n}{2} - 2|E(\Gamma)|.$$

Definition 2.8. [22] Let Γ be a simple connected graph. The Harary index of Γ , denoted by $H(\Gamma)$, is defined as:

$$H(\Gamma) = \sum_{u, v \in V(\Gamma)} \frac{1}{d(u, v)},$$

where $d(u, v)$ is the distance between vertices u and v .

Theorem 2.9. [23] Let Γ be a simple connected graph with $|V(\Gamma)| = n$ and diameter $\text{diam}(\Gamma) \leq 2$. Then the Harary index of Γ is given by:

$$H(\Gamma) = \frac{1}{2} \left(|E(\Gamma)| + \binom{n}{2} \right).$$

Definition 2.10. [11] Let Γ be a simple connected graph. The Forgotten index of Γ , denoted by $F(\Gamma)$, is defined as:

$$F(\Gamma) = \sum_{v \in V(\Gamma)} (\deg(v))^3,$$

where $\deg(v)$ denotes the degree of the vertex v in Γ .

3. Result and Discussion

This section is divided into two main parts. The first part discusses the structure of the power graph associated with the dihedral group D_{2n} , where $n = pq$ and p and q are distinct odd prime numbers. The second part focuses on the connectivity indices derived from the structural patterns obtained in the first part.

3.1. Structure of the Power Graph of the Dihedral Group D_{2pq}

Before analyzing the structure of the power graph of the dihedral group, we first recall the definition of the power graph.

Definition 3.1. [3] Let G be a group and let $a, b \in G$. The power graph of G , denoted by $\Gamma(G)$, is the graph whose vertex set is G , where two distinct vertices a and b are adjacent if and only if there exists a natural number n such that $a^n = b$ or $b^n = a$.

Example 3.2. Let D_{30} be a dihedral group. The power graph of this group is represented by the graph shown in Figure 1.

From Figure 1, the degrees of the vertices can be determined as follows:

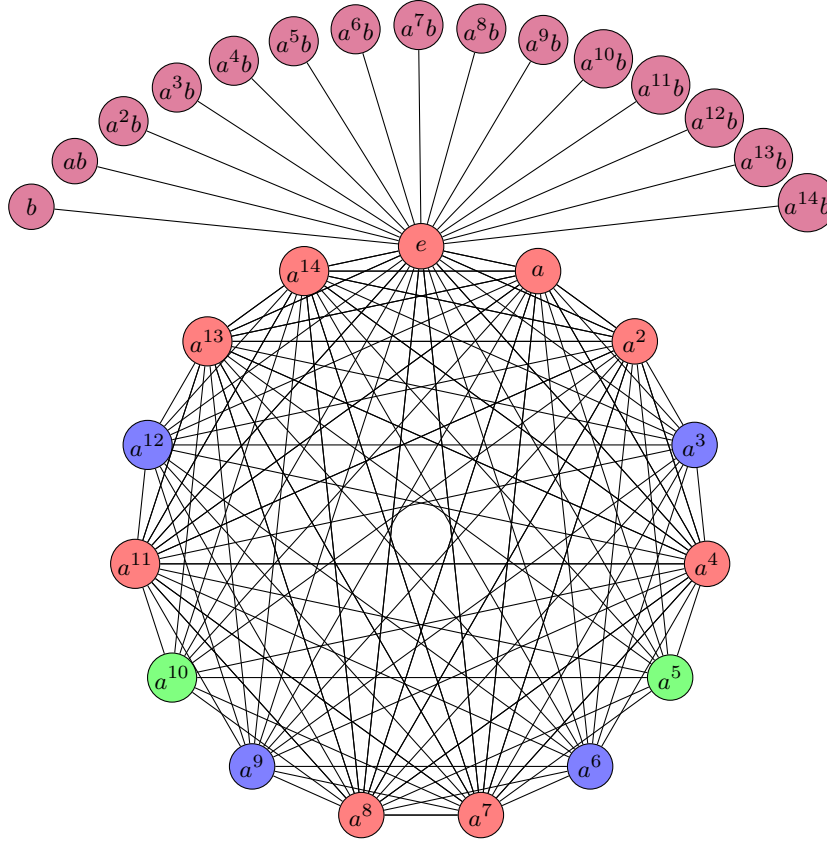
$$\begin{aligned} \deg(a) &= \deg(a^2) = \deg(a^4) = \deg(a^7) = \deg(a^8) \\ &= \deg(a^{11}) = \deg(a^{13}) = \deg(a^{14}) = 14, \\ \deg(a^3) &= \deg(a^6) = \deg(a^9) = \deg(a^{12}) = 12, \\ \deg(a^5) &= \deg(a^{10}) = 10, \\ \deg(e) &= 29, \quad \deg(a^i b) = 1, \end{aligned}$$

for $i = 0, 1, 2, \dots, 14$.

It follows that the vertex degrees in D_{30} can be classified into five distinct categories. The identity element connected to any vertices, so this vertices have degree 29. The generators of the group have degree 14. The vertices of order 5 have degree 12, whereas the vertices of order 3 have degree 10. In contrast, the vertices of order 2 have degree 1.

The pattern observed in Example 3.2 can be extended to a general dihedral group D_{2n} , where $n = pq$ is the product of two distinct odd prime numbers. For this purpose, the set of vertices of order p is denoted by V_p , those of order q by V_q , and the generators by V_{pq} . The identity vertex is denoted by V_e , while the vertices of order 2 are denoted by V_2 .

Using this notation, the degrees of the vertices in the power graph of the dihedral group D_{2n} with $n = pq$ are presented in the following theorem.

Figure 1. Power Graph of D_{30}

Theorem 3.3. Let $G = D_{2n}$ be a dihedral group of order $n = pq$. If p and q are distinct odd prime numbers, then for every $v \in V(\Gamma_G)$, we have:

$$\deg(v) = \begin{cases} 2pq - 1, & \text{for } v \in V_e, \\ pq - 1, & \text{for } v \in V_{pq}, \\ (p - 1)q, & \text{for } v \in V_q, \\ (q - 1)p, & \text{for } v \in V_p, \\ 1, & \text{for } v \in V_2. \end{cases}$$

Proof. Let $G = D_{2n}$ with $n = pq$, where p and q are distinct odd prime numbers. By Theorem 2.2, for any element $a^i b^j \in G$, its order is given by:

$$o(a^i b^j) = \begin{cases} \frac{n}{\gcd(i, n)}, & \text{if } j = 0, \\ 2, & \text{if } j = 1. \end{cases}$$

The degrees in the power graph Γ_G can be determined according to the order of each vertex. First, consider $v \in V_e$. Then $v = e$ and $o(e) = 1$. Since e is a power of

every element in G , it is adjacent to all other vertices. Hence,

$$\deg(e) = |G| - 1 = 2pq - 1.$$

Next, let $v \in V_{pq}$. Then $v = a^i$ with $\gcd(i, n) = 1$, and thus $o(v) = pq$. Hence, v generates the cyclic subgroup $\langle a \rangle$ of order pq , so it is adjacent to all elements in $\langle a \rangle$ except itself. Therefore,

$$\deg(v) = pq - 1.$$

Moreover, v is not adjacent to any element of the form $a^k b$, since such elements have order 2.

Now, let $v \in V_q$. Then $v = a^i$ with $\gcd(i, n) = p$, so $o(v) = q$. Hence, v is adjacent to all elements in the cyclic subgroup $\langle v \rangle$ except itself, contributing $q - 1$ vertices, and also to all elements in V_{pq} , since v is a power of each of them. Thus,

$$\deg(v) = (q - 1) + (p - 1)(q - 1) = (p - 1)q.$$

Similarly, let $v \in V_p$. Then $v = a^i$ with $\gcd(i, n) = q$, so $o(v) = p$. Hence, v is adjacent to all elements in $\langle v \rangle$ except itself, contributing $p - 1$ vertices, and also to all elements in V_{pq} . Therefore,

$$\deg(v) = (p - 1) + (p - 1)(q - 1) = (q - 1)p.$$

Finally, let $v \in V_2$. Then $v = a^i b$ for some i , and by the theorem $o(v) = 2$. Hence, $v^2 = e$, and the only element adjacent to v is e . Therefore for any $v \in V_2$, $\deg(v) = 1$. $\square$

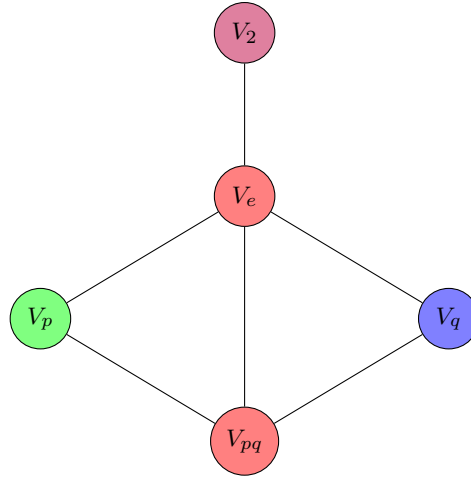
Based on the preceding discussion, the vertex set of the power graph of the dihedral group D_{2n} , where $n = pq$ with p and q distinct prime numbers, can be partitioned into mutually disjoint subsets as follows:

$$\begin{aligned} V_e &= \{e\}, \\ V_p &= \{a^q, a^{2q}, \dots, a^{(p-1)q}\}, \\ V_q &= \{a^p, a^{2p}, \dots, a^{(q-1)p}\}, \\ V_{pq} &= \{a^i \mid \gcd(i, n) = 1\}, \\ V_2 &= \{a^i b \mid i = 0, 1, 2, \dots, pq - 1\}. \end{aligned}$$

Clearly,

$$|V_{pq}| = (p - 1)(q - 1), \quad |V_p| = q - 1, \quad |V_q| = p - 1, \quad |V_e| = 1, \quad |V_2| = pq = n.$$

The adjacency relations among these subsets can be described as follows. The vertices in V_2 are adjacent only to the vertex in V_e . The vertices in V_{pq} are adjacent to all vertices in $V_e \cup V_p \cup V_q \cup V_{pq}$, but not to those in V_2 . Furthermore, each vertex in V_p is adjacent to all vertices in V_{pq} , and similarly, each vertex in V_q is adjacent to all vertices in V_{pq} . Finally, the vertex in V_e is adjacent to all vertices in $G \setminus \{e\}$. A schematic representation of these adjacency relations is illustrated in Figure 2.

Figure. 2. Compressed power graph of D_{2pq}

3.2. Connectivity Indices of Power Graph over Dihedral Group D_{2pq}

In this section, five connectivity indices of the power graph of the dihedral group D_{2pq} are investigated, based on the connectivity patterns established in the previous section. These indices include the First Zagreb index, the Wiener index, the hyper-Wiener index, the Harary index, and the Forgotten index.

Theorem 3.4. *Let $G = D_{2n}$ be a dihedral group with $n = pq$, where p and q are distinct odd prime numbers, and let $m = p + q$. Then, the First Zagreb index of the power graph Γ_G is given by:*

$$M_1(\Gamma_G) = n^3 - 3n^2 + 5nm - 2n - m^2 - m + 2.$$

Proof. Based on the degree classification of the vertices in Γ_G given in Theorem 3.3, the vertex is divided into V_e, V_{pq}, V_q, V_p , and V_2 . Thus,

$$\begin{aligned} M_1(\Gamma_G) &= \sum_{v \in V(\Gamma_G)} \deg(v)^2 \\ &= \sum_{v \in V_e} \deg(v)^2 + \sum_{v \in V_{pq}} \deg(v)^2 + \sum_{v \in V_q} \deg(v)^2 \\ &\quad + \sum_{v \in V_p} \deg(v)^2 + \sum_{v \in V_2} \deg(v)^2 \\ &= (2pq - 1)^2 + (p - 1)(q - 1)(pq - 1)^2 \\ &\quad + (p - 1)((p - 1)q)^2 + (q - 1)((q - 1)p)^2 + pq. \end{aligned}$$

Expanding and simplifying the above expression, we obtain:

$$M_1(\Gamma_G) = p^3q^3 - 3p^2q^2 + 5p^2q + 5pq^2 - 4pq - p^2 - q^2 - p - q + 2.$$

Since $n = pq$ and $m = p + q$, we rewrite each term as follows:

$$p^3q^3 = n^3, \quad p^2q^2 = n^2, \quad p^2q + pq^2 = n(p + q) = nm,$$

and

$$p^2 + q^2 = (p + q)^2 - 2pq = m^2 - 2n.$$

Substituting these into the expression yields:

$$\begin{aligned} M_1(\Gamma_G) &= n^3 - 3n^2 + 5nm - 4n - (m^2 - 2n) - m + 2 \\ &= n^3 - 3n^2 + 5nm - 2n - m^2 - m + 2. \end{aligned} \quad \square$$

Theorem 3.5. *Let $G = D_{2n}$ be a dihedral group with $n = pq$, where p and q are distinct odd prime numbers. Let $m = p + q$, and let Γ_G be the power graph of G . Then the Wiener index of Γ_G is given by:*

$$W(\Gamma_G) = \frac{7n^2 - 3n - 2m + 2}{2}.$$

Proof. From the structure of Γ_G , the distance between any two distinct vertices is at most 2. By Theorem 2.5,

$$W(\Gamma_G) = 2 \binom{|V(\Gamma_G)|}{2} - |E(\Gamma_G)|.$$

Since $|V(\Gamma_G)| = 2n$, we obtain:

$$2 \binom{2n}{2} = 2n(2n - 1) = 4n^2 - 2n.$$

Next, we compute $|E(\Gamma_G)|$. Based on the partition of vertices into V_e , V_{pq} , V_p , V_q , and V_2 , we have:

$$\begin{aligned} |E(\Gamma_G)| &= \frac{1}{2} \sum_{v \in V(\Gamma_G)} \deg(v), \\ &= \frac{1}{2} \left(\sum_{v \in V_e} \deg(v) + \sum_{v \in V_{pq}} \deg(v) + \sum_{v \in V_q} \deg(v) \right. \\ &\quad \left. + \sum_{v \in V_p} \deg(v) + \sum_{v \in V_2} \deg(v) \right), \\ &= \frac{1}{2} \left((2n - 1) + (p - 1)(q - 1)(n - 1), \right. \\ &\quad \left. + (p - 1)(p - 1)q + (q - 1)(q - 1)p + n \right), \\ &= \frac{1}{2} \left(3n - 1 + (p - 1)(q - 1)(n - 1) + q(p - 1)^2 + p(q - 1)^2 \right), \\ &= \frac{1}{2} (p^2q^2 - pq + 2p + 2q - 2), \\ &= \frac{1}{2} (n^2 - n + 2m - 2). \end{aligned}$$

Therefore,

$$\begin{aligned} W(\Gamma_G) &= 4n^2 - 2n - \frac{n^2 - n + 2m - 2}{2}, \\ &= \frac{7n^2 - 3n - 2m + 2}{2}. \end{aligned} \quad \square$$

Theorem 3.6. Let $G = D_{2n}$ be a dihedral group with $n = pq$, where p and q are distinct odd prime numbers. Let $m = p + q$, and let Γ_G be the power graph of G . Then the hyper-Wiener index of Γ_G is given by:

$$WW(\Gamma_G) = 5n^2 - 2n - 2m + 2.$$

Proof. Since the diameter of Γ_G is at most 2, we have $d(u, v) \in \{1, 2\}$ for any distinct vertices $u, v \in V(\Gamma_G)$. Hence, by the Theorem 2.7, the hyper-Wiener index satisfies:

$$WW(\Gamma_G) = 3 \binom{|V(\Gamma_G)|}{2} - 2|E(\Gamma_G)|.$$

Since $|V(\Gamma_G)| = 2n$, we obtain:

$$\binom{|V(\Gamma_G)|}{2} = \binom{2n}{2} = n(2n - 1).$$

Thus,

$$3 \binom{2n}{2} = 3n(2n - 1) = 6n^2 - 3n.$$

From the previous result,

$$|E(\Gamma_G)| = \frac{n^2 - n + 2m - 2}{2}.$$

Substituting these values, we get:

$$\begin{aligned} WW(\Gamma_G) &= 6n^2 - 3n - 2 \left(\frac{n^2 - n + 2m - 2}{2} \right), \\ &= 6n^2 - 3n - (n^2 - n + 2m - 2), \\ &= 5n^2 - 2n - 2m + 2. \end{aligned} \quad \square$$

Theorem 3.7. Let $G = D_{2n}$ be a dihedral group with $n = pq$, where p and q are distinct odd prime numbers. Let $m = p + q$, and let Γ_G be the power graph of G . Then the Harary index of Γ_G is given by:

$$H(\Gamma_G) = \frac{5n^2 - 3n + 2m - 2}{4}.$$

Proof. Since the diameter of Γ_G is at most 2, we have $d(u, v) \in \{1, 2\}$ for any distinct vertices $u, v \in V(\Gamma_G)$. Hence, by the Theorem 2.9, the Harary index satisfies:

$$H(\Gamma_G) = \frac{1}{2} \left(|E(\Gamma_G)| + \binom{|V(\Gamma_G)|}{2} \right).$$

Since $|V(\Gamma_G)| = 2n$, the number of pairs of vertices is:

$$\binom{|V(\Gamma_G)|}{2} = \binom{2n}{2} = n(2n-1) = 2n^2 - n.$$

From the previous result for the number of edges in the power graph of a dihedral group D_{2pq} , we have:

$$|E(\Gamma_G)| = \frac{n^2 - n + 2m - 2}{2}.$$

Substituting these values into the Harary index formula, we obtain:

$$\begin{aligned} H(\Gamma_G) &= \frac{1}{2} \left(\frac{n^2 - n + 2m - 2}{2} + (2n^2 - n) \right), \\ &= \frac{1}{2} \left(\frac{n^2 - n + 2m - 2 + 2(2n^2 - n)}{2} \right), \\ &= \frac{5n^2 - 3n + 2m - 2}{4}. \end{aligned} \quad \square$$

Theorem 3.8. *Let $G = D_{2n}$ be a dihedral group with $n = pq$, where p and q are distinct odd prime numbers, and let $m = p + q$. Then, the Forgotten index of the power graph Γ_G is given by:*

$$F(\Gamma_G) = n^4 - 2n^3 + 9mn^2 - 4n^2 - 4nm^2 - 6nm + 8n + m^3 + m - 2.$$

Proof. Based on the degree classification of the vertices in Γ_G given in Theorem 3.3, the vertex set is divided into V_e, V_{pq}, V_q, V_p , and V_2 . Thus, by the definition of the Forgotten index, we have:

$$\begin{aligned} F(\Gamma_G) &= \sum_{v \in V(\Gamma_G)} \deg(v)^3, \\ &= \sum_{v \in V_e} \deg(v)^3 + \sum_{v \in V_{pq}} \deg(v)^3 + \sum_{v \in V_q} \deg(v)^3 + \sum_{v \in V_p} \deg(v)^3 + \sum_{v \in V_2} \deg(v)^3, \\ &= (2pq - 1)^3 + (p - 1)(q - 1)(pq - 1)^3 + (p - 1)((p - 1)q)^3, \\ &\quad + (q - 1)((q - 1)p)^3 + pq. \end{aligned}$$

By substituting $n = pq$, we expand each term separately. First, the term for the identity element is $(2n - 1)^3 = 8n^3 - 12n^2 + 6n - 1$. Second, the term for the vertices in V_{pq} is expanded as $(pq - (p + q) + 1)(n - 1)^3 = (n - m + 1)(n^3 - 3n^2 + 3n - 1)$. Third, the terms for V_q and V_p are combined as $(p - 1)^4 q^3 + (q - 1)^4 p^3$, which expands to:

$$\begin{aligned} (p - 1)^4 q^3 + (q - 1)^4 p^3 &= (p^4 - 4p^3 + 6p^2 - 4p + 1)q^3 + (q^4 - 4q^3 + 6q^2 - 4q + 1)p^3, \\ &= (p^4 q^3 + q^4 p^3) - 4(p^3 q^3 + q^3 p^3) + 6(p^2 q^3 + q^2 p^3), \\ &\quad - 4(pq^3 + qp^3) + (q^3 + p^3), \\ &= n^3(p + q) - 4(2n^3) + 6n^2(p + q), \\ &\quad - 4n(p^2 + q^2) + (m^3 - 3nm), \\ &= n^3 m - 8n^3 + 6n^2 m - 4n(m^2 - 2n) + m^3 - 3nm. \end{aligned}$$

Substituting these identities into the expression and combining them with the remaining terms from $(2n-1)^3$, $(n-m+1)(n-1)^3$, and the reflection vertices V_2 , we perform the final simplification. The final simplification of $F(\Gamma_G)$ is obtained by expanding each part and regrouping the terms as follows:

$$\begin{aligned}
 F(\Gamma_G) &= (8n^3 - 12n^2 + 6n - 1) + (n - m + 1)(n^3 - 3n^2 + 3n - 1) \\
 &\quad + (n^3m - 8n^3 + 6n^2m - 4nm^2 + 8n^2 + m^3 - 3nm) + n, \\
 &= (8n^3 - 12n^2 + 6n - 1) + (n^4 - 2n^3 - mn^3 + 3mn^2 - 3mn + 2n + m - 1) \\
 &\quad + (n^3m - 8n^3 + 6n^2m - 4nm^2 + 8n^2 + m^3 - 3nm) + n, \\
 &= n^4 + (8n^3 - 2n^3 - mn^3 + n^3m - 8n^3) \\
 &\quad + (-12n^2 + 3mn^2 + 6n^2m + 8n^2) \\
 &\quad + (6n + 2n - 3mn - 4nm^2 - 3nm + n) \\
 &\quad + (-1 - 1 + m + m^3), \\
 &= n^4 - 2n^3 + 9mn^2 - 4n^2 - 4nm^2 - 6nm + 9n + m^3 + m - 2. \quad \square
 \end{aligned}$$

4. Conclusion

This study has successfully investigated the structural properties and topological connectivity indices of the undirected power graph associated with the dihedral group D_{2n} , specifically for the composite order $n = pq$, where p and q are distinct odd prime numbers. By systematically partitioning the vertex set into five distinct subsets based on the algebraic orders of the group elements (V_e, V_{pq}, V_p, V_q , and V_2), the exact degrees of all vertices and the total number of edges within the graph were accurately determined. Utilizing these foundational structural patterns, this paper derived the exact analytical formulas for five prominent connectivity indices: the first Zagreb index, the Wiener index, the hyper-Wiener index, the Harary index, and the Forgotten index.

The results demonstrate that these topological indices can be elegantly expressed as explicit polynomial functions of the group's primary parameters, specifically $n = pq$ and $m = p + q$. For future research, this work can be naturally extended by investigating the power graphs of dihedral groups with more complex composite orders, such as $n = pqr$ or $n = p^2q$, or by exploring other recently introduced topological indices such as the Sombor index or the second Zagreb index.

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