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ON METRIC DIMENSION IN GENERALIZED PARACHUTE GRAPH AND DUTCH WINDMILL GRAPH CORONA NULL GRAPH

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Abstract. *Let G be a connected graph with vertex set $V(G)$ and edge set $E(G)$. The distance between two vertices v and x in G is defined as the length of the shortest path joining v and x, denoted by $d(v,x)$. For an ordered subset $Y = \{y_1, y_2, \cdots, y_k\}$ of vertices in a connected graph G, and any vertex $x \in V(G)$, the metric representation of x with respect to Y is the k-tuple $r(x|Y) = (d(x,y_1), d(x,y_2), \cdots, d(x,y_k))$. A subset Y is called a resolving set for G if $r(v|Y) = r(x|Y)$ implies $v = x$ for every pair of vertices $v, w \in V(G)$. The metric dimension of G, denoted by $\dim(G)$, is the minimum cardinality among all resolving sets of G. A generalized parachute graph is denoted by P_n^m where n represents the number of vertices in the path graph P_n and m is the length of the straight path added under the double fan graph. A Dutch windmill graph corona null graph $D_n^m \odot N_k$ with $n \geq 3$, $m \geq 2$, and $k \geq 1$ is a graph formed from one copy of the Dutch windmill graph D_n^m and $m(n-1)+1$ copy of the null graph N_k, which each vertex in the ith copy of the null graph is connected to the i^{th} vertex of the Dutch windmill graph. The objective of this paper is to determine the metric dimension of the generalized parachute graph and the Dutch windmill graph corona with a null graph. The method employed in this study is a literature review based on relevant books and journal articles concerning metric dimension. The results obtained are as follows: the metric dimension of the generalized parachute graph P_n^m with $m \geq 1$, $n \geq 2$ is $dim P_n^m = \lfloor \frac{2n+7}{5} \rfloor$, and the metric dimension of the Dutch windmill graph corona null graph $D_n^m \odot N_k$ with $n \geq 3$, $m \geq 2$, and $k \geq 1$ is $dim(D_n^m \odot N_k) = (k-1)(m(n-1)+1)$. It can be concluded that the metric dimension of the generalized parachute graph and the Dutch windmill graph corona with a null graph is determined by the number of vertices in the constituent graphs.*

Keywords: dutch windmill graph corona null graph, generalized parachute graph; metric dimension, resolving set

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1. Introduction

One area of mathematics that can be used to solve various real-world problems is graph theory. A graph G is an ordered pair $(V(G), E(G))$, where $V(G) = \{v_1, v_2, \cdots, v_n\}$ is a finite nonempty set called the vertex set, and $E(G) = \{e_1, e_2, \cdots, e_m\}$ is a set of unordered pairs of elements of $V(G)$, called edges. One concept in graph theory that has recently attracted attention is the metric dimension.

The concept of metric dimensions emerged from the resolving set introduced by Slater in 1975 as well as by Harary and Melter in 1976 [1]. [2] defines a set Y as a set of vertices in a graph G such that for each vertex in G had a distinct distance to each vertex in Y. [2] introduces the concept of this set with the term locating set, whereas [3] introduced this set with the term resolving set [1].

For an ordered subset $Y = \{y_1, y_2, \cdots, y_k\}$ of vertices in a connected graph G, and a vertex $x \in V(G)$, the metric representation of x relative to Y is the k-tuple $r(x|Y) = (d(x, y_1), d(x, y_2), \cdots, d(x, y_k))$. The subset Y is a resolving set of G if the equality $r(v|Y) = r(x|Y)$ implies $v = x$ for every pair of vertices $v, x \in V(G)$. The metric dimension of G, denoted by $\dim(G)$, is the smallest cardinality of a resolving set of G [1].

In 2000, [1] proved that a connected graph G with order n has a metric dimension $dim(G) = 1$ if and only if $G = P_n$. Furthermore, they also established that a connected graph G of order $n \geq 2$ has metric dimension $\dim(G) = n - 1$ if and only if $G = K_n$. In 2012, [4] proved the metric dimension of generalized Petersen graphs. In 2021, [5] proved the metric dimension of Dutch windmill graph. In 2023, [6] showed that the metric dimension of the double fan graph $2K_1 + P_n$ is 3.

Many researchers have applied the concept of metric dimension to certain classes of graphs. In 2000, [1] studied the metric dimension of path graph P_n and complete graph K_n with n vertices, with the result that $dim(G) = 1$ if and only if $G = P_n$ $(n \geq 2)$ and $dim(G) = n - 1$ if and only if $G = K_n$ $(n \geq 2)$. In 2003, [7] determined the metric dimension of the wheel graph W_n, showing that $\dim(G) = 3$ for $n = 3, 6$ and $\dim(G) = \left\lfloor \frac{2n+2}{5} \right\rfloor$ for all other values of n. Furthermore, in 2019, [8] investigated the metric dimension under amalgamation operations on sunflower graphs, lollipop graphs, as well as caveman graphs. In 2019, [9] found the metric dimension of the generalized broken fan graph and the results of edge corona operations on path graphs and star graphs. Then in 2020, [10] found the metric dimension of the starbarbell graph and the results of edge corona operations on cycle graphs and path graphs.

Furthermore, recent studies from 2025 on the local metric dimension of k-multilevel corona graphs show that the obtained values depend strongly on the structure of the component graphs. In addition, this operation is not commutative, so $G \odot^k H$ is not necessarily isomorphic to $H \odot^k G$ and may yield different local metric dimension values [11]. In general, determining the metric dimension for any graph is very difficult. [12] and also [13] proved that finding the metric dimension of any graph is an NP-complete problem. Even the problem is still NP-complete if applied to certain classes of graphs, such as bipartite graphs [14], or planar graphs

[15]. These studies are the basis of reference in this study on other graph classes. The graph classes that will be studied in this paper are graph classes of generalized parachute graph and Dutch windmill graph corona null graph.

2. Results and Discussion

2.1. *Generalized Parachute Graph*

A generalized parachute graph is denoted by P_n^m, where n is the number of vertices in the path graph P_n and m is the length of the straight path added under the double fan graph [16]. A generalized parachute graph P_n^m has a set of vertices $V(P_n^m) = \{u_1, u_2, x_i, y_j \mid 2 \leq i \leq n;\ 1 \leq j \leq m\}$ with $|V(P_n^m)| = n + m + 2$ and $|E(P_{nm})| = 3n + m - 1$.

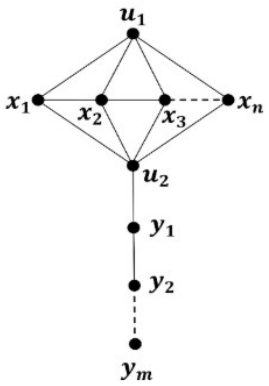


Figure. 1. Generalized Parachute Graph P_n^m

Theorem 2.1. *If P_n^m is a generalized parachute graph with $m \geq 1$, $n \geq 2$, then*

$$\dim(P_n^m) = \left\lfloor \frac{2n+7}{5} \right\rfloor.$$

Proof. Let P_n^m be a generalized parachute graph with $V(P) = V(U) \cup V(X) \cup V(Y)$, where $V(U) = \{u_1, u_2\}$, $V(X) = \{x_1, x_2, \cdots, x_n\}$, and $V(Y) = \{y_1, y_2, \cdots, y_m\}$. If the vertex set is taken from $V(Y)$, then the representation obtained will be the same for each vertex in $V(X)$. However, if the vertex set is taken from $V(U)$ and $V(X)$, then the representation obtained has a unique value for each vertex in $V(Y)$ so that the vertex set is a resolving set. Thus, $V(Y)$ is not included in the resolving set on the generalized parachute graph.

If we take any vertex $x_t \in V(X)$, $t \in [1, n]$, then the representation of vertex x_t to every vertex in $V(X)$ is 2, except for vertices x_{t-1}, x_t, and x_{t+1}, i.e., each has a value of 1, 0, and 1. Because there cannot exist two different vertices that have the same representation value, we look at the patterns for distance between pairs of vertices that have values of 1 and 0.

There are five cases to prove the metric dimension of a generalized parachute graph P_n^m according to the value of n.

(1) Case for $n \equiv 0 \pmod 5$

(i) Let $W = \{u_1, x_s\}$ where $s = 2, 4, 7, 9, \cdots, n-6, n-3, n-1$. Suppose $|W| < \lfloor \frac{2n+7}{5} \rfloor$. Then there exists $V = \{u_1\} \cup \{x_s\}$ which is not contained in W, such that there exist $u, v \in V(P_{nm})$ with $r(u|W) = r(v|W)$, which is a contradiction. Thus, $dim(P_n^m) \geq \lfloor \frac{2n+7}{5} \rfloor$.

(ii) Now let $W = \{u_1, x_s\}$ where $s = 2, 4, 7, 9, \cdots, n-6, n-3, n-1$. Then the representation obtained has a unique value for each vertex in $V(P_n^m)$. Hence, W is a basis with $|W| = \lfloor \frac{2n+7}{5} \rfloor$. Therefore, $dim(P_n^m) = \lfloor \frac{2n+7}{5} \rfloor$.

(2) Case for $n \equiv 1 \pmod 5$

(i) Let $W = \{u_1, x_s\}$ where $s = 2, 4, 7, 9, \cdots, n-7, n-4, n-2$. Suppose $|W| < \lfloor \frac{2n+7}{5} \rfloor$. Then there exists $V = \{u_1\} \cup \{x_s\}$ which is not contained in W, such that there exist $u, v \in V(P_n^m)$ with $r(u|W) = r(v|W)$, which is a contradiction. Thus, $dim(P_n^m) \geq \lfloor \frac{2n+7}{5} \rfloor$.

(ii) Now let $W = \{u_1, x_s\}$ where $s = 2, 4, 7, 9, \cdots, n-7, n-4, n-2$. Then the representation obtained has a unique value for each vertex in $V(P_n^m)$. Hence, W is a basis with $|W| = \lfloor \frac{2n+7}{5} \rfloor$. Therefore, $dim(P_n^m) = \lfloor \frac{2n+7}{5} \rfloor$.

(3) Case for $n \equiv 2 \pmod 5$

(i) Let $W = \{u_1, x_s\}$ where $s = 2, 4, 7, 9, \cdots, n-5, n-3, n$. Suppose $|W| < \lfloor \frac{2n+7}{5} \rfloor$. Then there exists $V = \{u_1\} \cup \{x_s\}$ which is not contained in W, such that there exist $u, v \in V(P_n^m)$ with $r(u|W) = r(v|W)$, which is a contradiction. Thus, $dim(P_n^m) \geq \lfloor \frac{2n+7}{5} \rfloor$.

(ii) Now let $W = \{u_1, x_s\}$ where $s = 2, 4, 7, 9, \cdots, n-5, n-3, n$. Then the representation obtained has a unique value for each vertex in $V(P_n^m)$. Hence, W is a basis with $|W| = \lfloor \frac{2n+7}{5} \rfloor$. Therefore, $dim(P_n^m) = \lfloor \frac{2n+7}{5} \rfloor$.

(4) Case for $n \equiv 3 \pmod 5$

(i) Let $W = \{u_1, x_s\}$ where $s = 2, 4, 7, 9, \cdots, n-6, n-4, n-1$. Suppose $|W| < \lfloor \frac{2n+7}{5} \rfloor$. Then there exists $V = \{u_1\} \cup \{x_s\}$ which is not contained in W, such that there exist $u, v \in V(P_n^m)$ with $r(u|W) = r(v|W)$, which is a contradiction. Thus, $dim(P_n^m) \geq \lfloor \frac{2n+7}{5} \rfloor$.

(ii) Now let $W = \{u_1, x_s\}$ where $s = 2, 4, 7, 9, \cdots, n-6, n-4, n-1$. Then the representation obtained has a unique value for each vertex in $V(P_n^m)$. Hence, W is a basis with $|W| = \lfloor \frac{2n+7}{5} \rfloor$. Therefore, $dim(P_n^m) = \lfloor \frac{2n+7}{5} \rfloor$.

(5) Case for $n \equiv 4 \pmod 5$

(i) Let $W = \{u_1, x_s\}$ where $s = 2, 4, 7, 9, \cdots, n-5, n-2, n$. Suppose $|W| < \lfloor \frac{2n+7}{5} \rfloor$. Then there exists $V = \{u_1\} \cup \{x_s\}$ which is not contained in W, such that there exist $u, v \in V(P_n^m)$ with $r(u|W) = r(v|W)$, which is a contradiction. Thus, $dim(P_n^m) \geq \lfloor \frac{2n+7}{5} \rfloor$.

(ii) Now let $W = \{u_1, x_s\}$ where $s = 2, 4, 7, 9, \cdots, n-5, n-2, n$. Then the representation obtained has a unique value for each vertex in $V(P_n^m)$. Hence, W is a basis with $|W| = \lfloor \frac{2n+7}{5} \rfloor$. Therefore, $dim(P_n^m) = \lfloor \frac{2n+7}{5} \rfloor$.

From the five cases above, it is proven that if P_n^m is a generalized parachute

graph with $m \geq 1$, $n \geq 2$, then

$$\dim(P_n^m) = \left\lfloor \frac{2n+7}{5} \right\rfloor. \qquad \square$$

The above result complements previous studies on the metric dimension of fan-related graph families. [6] proved that the metric dimension of the double fan graph $2K_1+P_n$ is equal to 3. In contrast, the metric dimension of the generalized parachute graph is expressed by the formula $\left\lfloor \frac{2n+7}{5} \right\rfloor$, which depends on the graph order. Another graph family considered in this study is the Dutch windmill graph corona null graph.

2.2. *Dutch Windmill Graph Corona Null Graph*

The Dutch windmill graph D_n^m, with $n \geq 3$ and $m \geq 2$, is defined as the graph formed by taking m copies of the cycle graph C_n that share a common central vertex [5]. A Dutch windmill graph corona null graph $D_n^m \odot N_k$ where $n \geq 3$, $m \geq 2$, and $k \geq 1$ is a graph formed from one copy of the Dutch windmill graph D_n^m and $m(n-1)+1$ multiple instances of the null graph N_k, where each vertex in the i^{th} instance of the null graph is connected to the i^{th} vertex of the Dutch windmill graph. A Dutch windmill graph corona null graph $D_n^m \odot N_k$ has $(mn-m+1)(k+1)$ vertices and $mnk+mn+k$ edges.

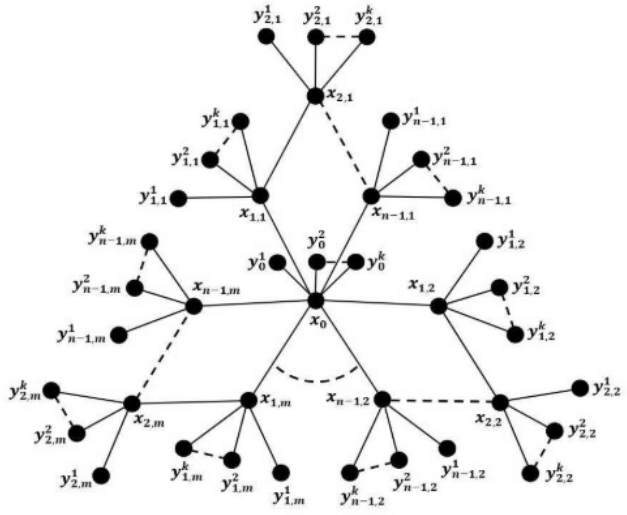


Figure. 2. Dutch windmill graph corona null graph $D_n^m \odot N_k$

Theorem 2.2. *If $D_n^m \odot N_k$ is a Dutch windmill graph corona null graph where $n \geq 3$, $m \geq 2$, and $k \geq 1$, then*

$$\dim(D_n^m \odot N_k) = (k-1)(n-1(m+1)).$$

Proof. Let $D_n^m \odot N_k$ is a Dutch windmill graph corona null graph where $n \geq 3$, $m \geq 2$, and $k \geq 1$.

(1) Let $W = \{y_{0t}, y_{r,st}\}$ where $r = 1, 2, \cdots, n-1; s = 1, 2, \cdots, m$; and $t = 1, 2, \cdots, k-1$, is a basis of a Dutch windmill graph corona null graph $D_n^m \odot N_k$ where $|W| < (k-1)(n-1(m+1))$. So there exists $V = y_{0t} \cup \{y_{r,st}\}$ which is not contained in W. Such that there exists $u, v \in V(D_n^m \odot N_k)$ where $r(u|W) = r(v|W)$ which is a contradiction. Thus $dim(D_n^m \odot N_k) \geq (k-1)(n-1(m+1))$.

(2) Let $W = \{y_{0t}, y_{r,st}\}$ where $r = 1, 2, \cdots, n-1; s = 1, 2, \cdots, m$; and $t = 1, 2, \cdots, k-1$, then the representation obtained has a unique value for each vertex in $V(D_n^m \odot N_k)$. So W is a basis, where $|W| \leq (k-1)(n-1(m+1))$. Thus $dim(D_n^m \odot N_k) \leq (k-1)(n-1(m+1))$.

Therefore, it is proven that if $D_n^m \odot N_k$ is a Dutch windmill graph corona null graph where $n \geq 3$, $m \geq 2$, and $k \geq 1$, then $dim(D_n^m \odot N_k) = (k-1)(n-1(m+1))$. □

By considering the main results as stated in Theorems 2.1 and Theorem 2.2, it appears that to find the metric dimension of a certain class of graphs, there is clearly no algorithm that can determine the metric dimension of a graph. This is in accordance with the opinion of [13] which proved that finding the metric dimension of a graph is an NP-complete problem. Moreover, this problem remains NP-complete even when restricted to certain classes of graphs.

For future work, it is suggested that readers explore the determination of metric dimensions for graph classes that have not yet been examined by previous researchers.

3. Conclusion

It follows that the metric dimension of both the graph classes of generalized the parachute graph as well as the Dutch windmill graph corona null graph are determined by the number of vertices in the constituent graphs.

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