

A COMPUTATIONAL APPROACH TO DETERMINING THE LOCATING-CHROMATIC NUMBER OF PIZZA GRAPHS

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Abstract. *This study investigates the locating-chromatic number ($L-cn$) of the pizza graph P_{zs} using a computational approach implemented in Python. For $s = 3, 4, 5, 6, 7$, and 8, the program constructs the corresponding pizza graphs and determines their locating colorings. The computational results show that the locating-chromatic number of all tested pizza graphs is 4. These findings confirm previously reported results for $s = 3$ and $s = 4$, while different values are obtained for $s = 5, 6, 7$, and 8. In addition, the execution time increases significantly as the graph order grows. The results provide computational evidence for the locating-chromatic properties of pizza graphs and motivate further theoretical investigation.*

Keywords: locating-chromatic number($L - cn$), pizza graph, Python programming, computational approach.

1. Introduction

Graph theory is an important branch of discrete mathematics with numerous applications in computer science, communication networks, optimization, transportation systems, and social network analysis. In recent years, graph coloring has attracted considerable attention due to both its theoretical significance and practical applications in scheduling, frequency assignment, artificial intelligence, and network design [1,2].

One important development in graph coloring is the locating-chromatic number, introduced by Chartrand *et al.* [3] based on the concept of partition dimension proposed by Chartrand *et al.* [4]. This parameter combines proper coloring and vertex

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identification through color codes, making it useful for distinguishing vertices in a graph. Since its introduction, the locating-chromatic number has been investigated for various graph classes, including disconnected graphs [5], origami graphs and their operations [6,7,8,9], corona operations of paths and cycles [10], rose graphs [11], sunflower graphs [12], and $(A, 4, n)$ graphs [13]. Furthermore, Irawan *et al.* [14] developed a computational procedure for determining the locating-chromatic number of origami graphs, demonstrating the effectiveness of computational methods in locating coloring problems.

Among the graph families that have attracted attention is the pizza graph, introduced by Nabila and Salman [15]. The pizza graph consists of a central vertex, intermediate vertices, and an outer cycle, forming a structure that combines radial and cyclic characteristics. This distinctive structure makes the pizza graph an interesting object for studying various graph-theoretical parameters.

Studies on the locating-chromatic number of pizza graphs remain limited. Surbakti *et al.* [16] determined the locating-chromatic number of pizza graphs, while Irawan and Asmiati [17] investigated the locating-chromatic number of certain barbell operations on pizza graphs and their subdivisions. In particular, Surbakti *et al.* [16] showed that the locating-chromatic number of P_{z_s} is 4 for $s = 3$ and s for $s \geq 4$.

Motivated by the limited computational studies on pizza graphs, this research aims to develop a computational algorithm for determining the locating-chromatic number of the pizza graph P_{z_s} and to verify the theoretical results reported by Surbakti *et al.* [16]. Implemented in Python, the algorithm generates color assignments, computes color codes, and identifies locating colorings of pizza graphs. The obtained results are then compared with existing theoretical findings to provide further insight into the locating-chromatic properties of pizza graphs.

2. Preliminaries

This section presents the basic concepts used in this study, namely the pizza graph and the locating-chromatic number.

Definition 2.1. [15] For an integer $s \geq 3$, the pizza graph P_{z_s} is defined by:

$$V(P_{z_s}) = \{x\} \cup \{y_i \mid 1 \leq i \leq s\} \cup \{z_i \mid 1 \leq i \leq s\},$$

and

$$E(P_{z_s}) = \{xy_i \mid 1 \leq i \leq s\} \cup \{y_iz_i \mid 1 \leq i \leq s\} \cup \{z_iz_{i+1} \mid 1 \leq i \leq s-1\} \cup \{z_s z_1\}.$$

The graph consists of a central vertex x , intermediate vertices $y_1, y_2, \dots, y_s$, and cycle vertices $z_1, z_2, \dots, z_s$.

Example 2.2. Figure 1 illustrates the pizza graph P_{z_s} for $s = 5$. The graph consists of one central vertex x , five intermediate vertices $y_1, y_2, \dots, y_5$, and five cycle vertices $z_1, z_2, \dots, z_5$. The central vertex x is adjacent to every intermediate vertex y_i , each intermediate vertex y_i is connected to the corresponding cycle vertex z_i , and the cycle vertices $z_1, z_2, \dots, z_5$ form a cycle. Hence, the graph contains

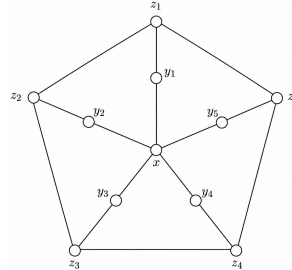


Figure. 1. An illustration of the pizza graph P_{z_s} with $s = 5$.

$2s + 1 = 11$ vertices and $3s = 15$ edges. Figure 1 provides a graphical illustration of this construction.

Definition 2.3. [3] Let $R = (V, E)$ be a connected graph and let h be a proper k -coloring of R . Suppose that:

$$\Pi = \{H_1, H_2, \dots, H_k\}$$

is the partition of $V(R)$ induced by h , where H_i denotes the set of vertices assigned the color i . For a vertex $v \in V(R)$, the color code of v with respect to Π is defined by:

$$h_{\Pi}(v) = (d(v, H_1), d(v, H_2), \dots, d(v, H_k)),$$

where

$$d(v, H_i) = \min\{d(v, u) : u \in H_i\}, \quad i = 1, 2, \dots, k.$$

If every pair of distinct vertices of R has a unique color code, then h is called a locating coloring of R . The minimum number of colors required to obtain a locating coloring of R is called the locating-chromatic number of R , denoted by $\chi_L(R)$.

Theorem 2.4. [16] Let P_{z_s} be a pizza graph. Then the locating-chromatic number of P_{z_s} is 4 for $s = 3$ and equals s for every $s \geq 4$.

3. Method

This study employs a computational approach to determine the locating-chromatic number of the pizza graph P_{z_s} . The procedure is implemented in Python. For a given value of s , the pizza graph P_{z_s} is first constructed according to Definition 2.1 [15]. The program then generates all possible color assignments, beginning with one color and increasing the number of colors until a valid locating coloring is found. For each assignment, the program verifies whether the coloring is proper, computes the color code of each vertex, and checks that all color codes are distinct. The minimum number of colors satisfying these conditions is recorded as the locating-chromatic number of P_{z_s} [4].

In addition to determining the locating-chromatic number, the program generates the corresponding color classes and color codes, visualizes the resulting locating coloring, and records the execution time required for each computational experiment. The computational experiments were conducted using the hardware and software specifications presented in Table 1.

Table 1. Hardware and software specifications

Item	Specification
Processor	Intel(R) Core(TM) 5 120U (12 CPUs), 1.40 GHz
Architecture	64-bit
Memory	16 GB RAM
Operating System	Windows 11
Python Version	Python 3.12.13
Libraries	Jupyter Notebook 6.5.7, NetworkX 3.6.1, IPython 7.34.0

Algorithm 1 summarizes the procedure for determining $\chi_L(P_{z_s})$.

4. Result and Discussion

To evaluate the performance of the proposed computational approach, a series of experiments was conducted on pizza graphs P_{z_s} for $s = 3, 4, 5, 6, 7$, and 8. For each instance, the program computes the locating-chromatic number and measures the time required to obtain a valid locating coloring. The resulting number of vertices, execution times, and locating-chromatic numbers are presented in Table 2.

Table 2. Measurement Results.

Cases	Type of P_{z_s}	Number of vertices	Execution Time	L-cn
1	P_{z_3}	7	0.0169 s	4
2	P_{z_4}	9	0.2426 s	4
3	P_{z_5}	11	1.1899 s	4
4	P_{z_6}	13	17.6600 s	4
5	P_{z_7}	15	257.5071 s	4
6	P_{z_8}	17	3890.0248 s	4

Table 2 presents the computational results obtained using the proposed algorithm for the pizza graphs P_{z_s} with $s = 3, 4, 5, 6, 7$, and 8. For each test case, the table reports the graph type, the number of vertices, the execution time, and the corresponding locating-chromatic number. The results show that the locating-chromatic number remains constant, namely $\chi_L(P_{z_s}) = 4$, for all tested values of s . Therefore, increasing the order of the graph within the tested range does not affect the minimum number of colors required to obtain a locating coloring.

Algorithm 1 Determining the Locating-Chromatic Number of P_{z_s}

Require: Integer $s \geq 3$

Ensure: $\chi_L(P_{z_s})$, color classes, color codes, execution time, and graph visualization

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1: Construct the pizza graph  $P_{z_s}$ 
2: Start timer
3:  $V \leftarrow V(P_{z_s})$ 
4: for  $c \leftarrow 1$  to  $|V|$  do
5:     Generate all color assignments using  $c$  colors
6:     for each coloring assignment  $h$  do
7:         if  $h$  is not a proper coloring then
8:             Continue to the next coloring assignment
9:         end if
10:    Determine the color classes
        
$$\Pi = \{H_1, H_2, \dots, H_c\}$$

11:    for each vertex  $v \in V$  do
12:        Compute the color code
            
$$h_\Pi(v) = (d(v, H_1), d(v, H_2), \dots, d(v, H_c))$$

13:    end for
14:    if two vertices have identical color codes then
15:        Continue to the next coloring assignment
16:    end if
17:     $\chi_L(P_{z_s}) \leftarrow c$ 
18:    Compute execution time
19:    Output color classes
20:    Output color codes
21:    Draw the graph  $P_{z_s}$ 
22:    return  $\chi_L(P_{z_s})$ , color classes, color codes, execution time, and graph
        visualization
23:    end for
24: end for

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Although the locating-chromatic number remains unchanged, the execution time increases considerably as the graph size grows. Specifically, the running time increases from only 0.0169 seconds for P_{z_3} to 3890.0248 seconds for P_{z_8} , indicating a rapid increase in computational cost. This behavior is expected because the proposed algorithm employs an exhaustive-search strategy that generates all candidate proper colorings and verifies the locating-coloring condition. Consequently, the size of the search space grows rapidly as the number of vertices increases.

Furthermore, the computational results agree with the theorem of Surbakti et al. [16] for the cases $s = 3$ and $s = 4$, thereby confirming the correctness of the proposed algorithm for graph instances with established theoretical results. However, for $s = 5, 6, 7$, and 8 , the algorithm consistently yields $\chi_L(P_{z_s}) = 4$, which differs

from the previously reported results. This discrepancy suggests that the existing theoretical results should be reexamined and demonstrates the value of computational approaches in verifying and validating graph-theoretical results.

The substantial increase in execution time also reveals the computational limitations of exhaustive-search algorithms when applied to larger graph instances. Therefore, future research may focus on developing more efficient algorithms by exploiting graph symmetries, pruning techniques, or structural properties of pizza graphs to reduce the search space while preserving correctness. Such improvements would enable the computation of the locating-chromatic number for pizza graphs of considerably larger orders.

To further illustrate the computational performance of the proposed algorithm, the execution times for all tested pizza graphs are presented in Figure 2. The figure clearly shows that the computational cost increases rapidly as the order of the pizza graph increases.

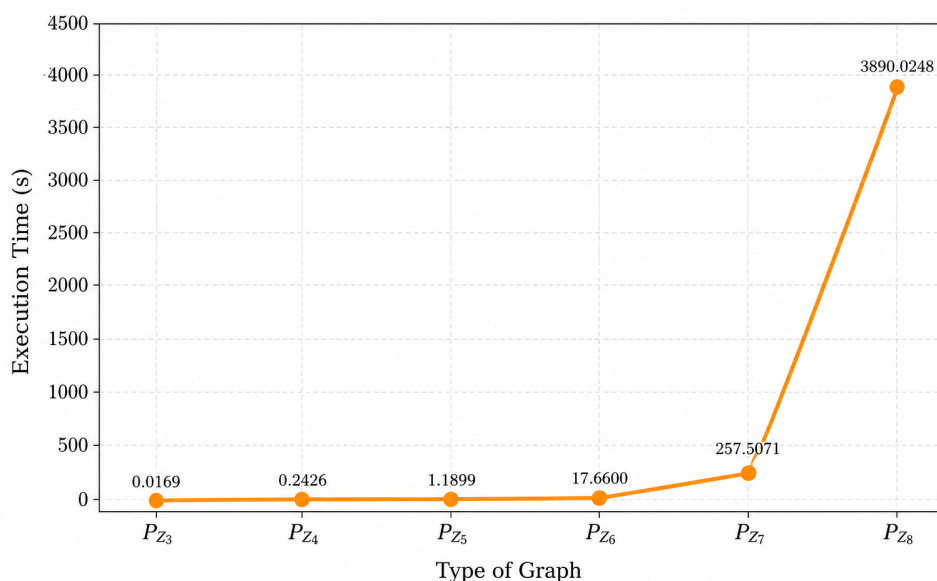


Figure 2. Execution time of P_{z_s} for $s = 3, 4, 5, 6, 7$, and 8 .

Figure 2 illustrates the execution time required to determine the locating-chromatic number of the pizza graph P_{z_s} for several values of s . The curve shows a substantial increase in execution time as the graph order increases. For smaller graphs, such as P_{z_3} and P_{z_4} , the program completes the computation in less than 1 second. However, the execution time grows rapidly for larger graphs, reaching 257.5071 seconds for P_{z_7} and 3890.0248 seconds for P_{z_8} .

This behavior is mainly attributed to the brute-force approach employed in the computational process. As the value of s increases, the number of vertices and the number of possible color assignments increase significantly, resulting in a much

larger search space that must be examined to identify a valid locating coloring. Consequently, the computational complexity increases considerably, leading to a rapid growth in execution time.

Despite the substantial increase in computational cost, the locating-chromatic number of all tested pizza graphs remains 4. This result indicates that the increase in execution time is due to the complexity of the search process rather than to an increase in the locating-chromatic number itself. Therefore, while the proposed algorithm is effective for determining the locating-chromatic number of pizza graphs, more efficient computational methods may be required for graphs of larger order.

The following results present $\chi_L(P_{z_s})$ for $s = 3, 4, 5, 6, 7$, and 8, obtained using Python.

Case 1. $s = 3$

The partition of $V(Pz_3)$ is :

$$H_1 = \{x, z_1\};$$

$$H_2 = \{y_1, y_2, y_3\};$$

$$H_3 = \{z_2\};$$

$$H_4 = \{z_3\}.$$

The color codes of the vertices are presented as follows:

$$h_{\Pi}(x) = (0, 1, 2, 2); h_{\Pi}(y_1) = (1, 0, 2, 2); h_{\Pi}(y_2) = (1, 0, 1, 2); h_{\Pi}(y_3) = (1, 0, 2, 1); h_{\Pi}(z_1) = (0, 1, 1, 1); h_{\Pi}(z_2) = (1, 1, 0, 1); h_{\Pi}(z_3) = (1, 1, 1, 0).$$

Case 2. $s = 4$

The partition of $V(Pz_4)$ is :

$$H_1 = \{x, z_1\};$$

$$H_2 = \{y_1, y_2, y_3\};$$

$$H_3 = \{z_2, z_4\};$$

$$H_4 = \{z_3, y_4\}.$$

The color codes of the vertices are presented as follows:

$$h_{\Pi}(x) = (0, 1, 2, 1); h_{\Pi}(y_1) = (1, 0, 2, 2); h_{\Pi}(y_2) = (1, 0, 1, 2); h_{\Pi}(y_3) = (1, 0, 2, 1); h_{\Pi}(y_4) = (1, 2, 1, 0); h_{\Pi}(z_1) = (0, 1, 1, 2); h_{\Pi}(z_2) = (1, 1, 0, 1); h_{\Pi}(z_3) = (2, 1, 1, 0); h_{\Pi}(z_4) = (1, 2, 0, 1).$$

Case 3. $s = 5$

The partition of $V(Pz_5)$ is :

$$H_1 = \{x, z_1, z_3\};$$

$$H_2 = \{y_1, y_2, y_3, y_4, y_5\};$$

$$H_3 = \{z_2, z_4\};$$

$$H_4 = \{z_5\}.$$

The color codes of the vertices are presented as follows:

$$h_{\Pi}(x) = (0, 1, 2, 2); h_{\Pi}(y_1) = (1, 0, 2, 2); h_{\Pi}(y_2) = (1, 0, 1, 3); h_{\Pi}(y_3) = (1, 0, 2, 3); h_{\Pi}(y_4) = (1, 0, 1, 2); h_{\Pi}(y_5) = (1, 0, 2, 1); h_{\Pi}(z_1) = (0, 1, 1, 1); h_{\Pi}(z_2) = (1, 1, 0, 2); h_{\Pi}(z_3) = (0, 1, 1, 2); h_{\Pi}(z_4) = (1, 1, 0, 1); h_{\Pi}(z_5) = (1, 1, 1, 0).$$

Case 4. $s = 6$

The partition of $V(Pz_6)$ is :

$$H_1 = \{x, z_1\};$$

$$H_2 = \{y_1, y_2, y_3, z_4, z_6\};$$

$$H_3 = \{z_2, y_4, z_5\};$$

$$H_4 = \{z_3, y_5, y_6\}.$$

The color codes of the vertices are presented as follows:

$$\begin{aligned} h_{\Pi}(x) &= (0, 1, 1, 1); h_{\Pi}(y_1) = (1, 0, 2, 2); h_{\Pi}(y_2) = (1, 0, 1, 2); h_{\Pi}(y_3) = \\ &= (1, 0, 2, 1); h_{\Pi}(y_4) = (1, 1, 0, 2); h_{\Pi}(y_5) = (1, 2, 1, 0); h_{\Pi}(y_6) = (1, 1, 2, 0); \\ h_{\Pi}(z_1) &= (0, 1, 1, 2); h_{\Pi}(z_2) = (1, 1, 0, 1); h_{\Pi}(z_3) = (2, 1, 1, 0); h_{\Pi}(z_4) = \\ &= (2, 0, 1, 1); h_{\Pi}(z_5) = (2, 1, 0, 1); h_{\Pi}(z_6) = (1, 0, 1, 1). \end{aligned}$$

Case 5. $s = 7$

The partition of $V(Pz_7)$ is :

$$H_1 = \{x, z_1, z_3, z_6\};$$

$$H_2 = \{y_1, y_2, y_3, y_4, y_5, y_6, y_7\};$$

$$H_3 = \{z_2, z_4\};$$

$$H_4 = \{z_5, z_7\}.$$

The color codes of the vertices are presented as follows:

$$\begin{aligned} h_{\Pi}(x) &= (0, 1, 2, 2); h_{\Pi}(y_1) = (1, 0, 2, 2); h_{\Pi}(y_2) = (1, 0, 1, 3); h_{\Pi}(y_3) = \\ &= (1, 0, 2, 3); h_{\Pi}(y_4) = (1, 0, 1, 2); h_{\Pi}(y_5) = (1, 0, 2, 1); h_{\Pi}(y_6) = (1, 0, 3, 2); \\ h_{\Pi}(y_7) &= (1, 0, 3, 1); h_{\Pi}(z_1) = (0, 1, 1, 1); h_{\Pi}(z_2) = (1, 1, 0, 2); h_{\Pi}(z_3) = \\ &= (0, 1, 1, 2); h_{\Pi}(z_4) = (1, 1, 0, 1); h_{\Pi}(z_5) = (1, 1, 1, 0); h_{\Pi}(z_6) = (0, 1, 2, 1); \\ h_{\Pi}(z_7) &= (1, 1, 2, 0). \end{aligned}$$

Case 6. Pizza Graph Pz_8

The partition of $V(Pz_8)$ is :

$$H_1 = \{x, z_1, z_3, z_5\};$$

$$H_2 = \{y_1, y_2, y_6, z_8\};$$

$$H_3 = \{z_2, y_3, y_4, z_7\};$$

$$H_4 = \{z_4, y_5, z_6, y_7, y_8\}.$$

The color codes of the vertices are presented as follows:

$$\begin{aligned} h_{\Pi}(x) &= (0, 1, 1, 1); h_{\Pi}(y_1) = (1, 0, 2, 2); h_{\Pi}(y_2) = (1, 0, 1, 2); h_{\Pi}(y_3) = \\ &= (1, 2, 0, 2); h_{\Pi}(y_4) = (1, 2, 0, 1); h_{\Pi}(y_5) = (1, 2, 2, 0); h_{\Pi}(y_6) = (1, 0, 2, 1); \\ h_{\Pi}(y_7) &= (1, 2, 1, 0); h_{\Pi}(y_8) = (1, 1, 2, 0); h_{\Pi}(z_1) = (0, 1, 1, 2); h_{\Pi}(z_2) = \\ &= (1, 1, 0, 2); h_{\Pi}(z_3) = (0, 2, 1, 1); h_{\Pi}(z_4) = (1, 3, 1, 0); h_{\Pi}(z_5) = (0, 2, 2, 1); \\ h_{\Pi}(z_6) &= (1, 1, 1, 0); h_{\Pi}(z_7) = (2, 1, 0, 1); h_{\Pi}(z_8) = (1, 0, 1, 1). \end{aligned}$$

The minimum locating colorings obtained from the computational procedure are illustrated in Figure 2. The figure presents the pizza graphs P_{z_s} for $s = 3, 4, 5, 6, 7$, and 8, together with their corresponding color classes. This visualization provides a clearer understanding of the coloring patterns and demonstrates how the locating coloring structure is maintained as the graph order increases.

Figure 3 shows that all tested pizza graphs admit a locating coloring using four

colors. Although the number of vertices increases as s grows, the same set of four color classes remains sufficient to distinguish all vertices through their color codes. This observation is consistent with the computational results presented in Table 2, where the locating-chromatic number is 4 for all tested values of s .

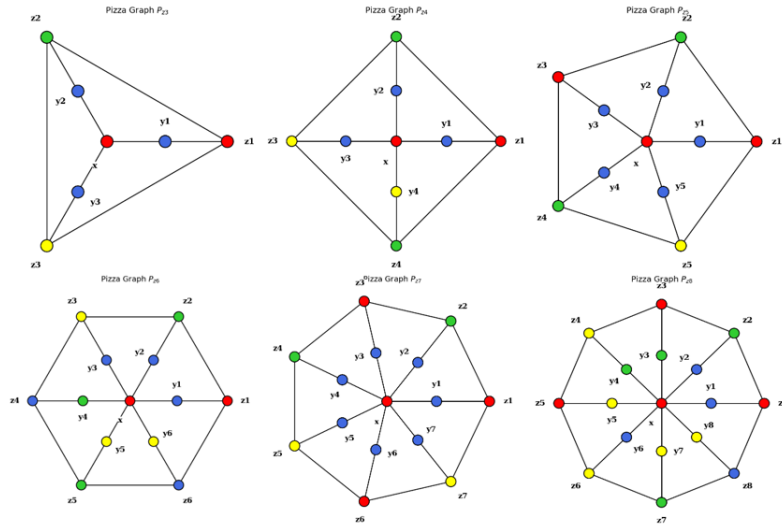


Figure. 3. Minimum locating colorings of P_{z_s} for $s = 3, 4, 5, 6, 7$, and 8 .

5. Conclusion

The computational results indicate that the locating-chromatic number of the pizza graph P_{z_s} is 4 for all tested values of s , namely $s = 3, 4, 5, 6, 7$, and 8 . The obtained results agree with the theorem of Surbakti *et al.* [16] for $s = 3$ and $s = 4$, but differ for $s = 5, 6, 7$, and 8 , where the computational approach yields $\chi_L(P_{z_s}) = 4$. Furthermore, the execution time increases substantially as the graph order grows. These findings suggest the need for further theoretical verification and computational investigation of the locating-chromatic number of pizza graphs.

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